Essays on Quantitative Macroeconomics

Dissertation
for the
Faculty of Economics, Business Administration
and Information Technology
of the
University of Zurich

to achieve the title of
Doctor of Philosophy
in Economics

presented by

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from China

approved in September 2010 at the request of

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The Faculty of Economics, Business Administration and Information Technology of the University of Zurich hereby authorizes the printing of this Doctoral Thesis, without thereby giving any opinion on the views contained therein.

Zurich, September 20, 2010

Chairman of the Doctoral Committee: Prof. Dr. Dieter Pfaff

To my mother

Preface

This thesis consists of three essays that I wrote during my Ph.D. studies. Chapters 1 and 3 are based on two solo-authored working papers. Chapter 2 is based on a working paper co-authored with Dr. Christoph Winter. The three essays are closely related under common themes. However, they are organized in a largely self-contained way, so that one can read each chapter individually. Technical details are provided in the appendixes, including analytical derivations and computational algorithms.

The lastest version of the paper in Chapter 1 is entitled "Underdevelopment of Financial Markets and Excess Consumption Growth Volatility in Developing Countries " and can be downloaded from

$https://sites.google.com/site/hengchenjmk/jobmarketpaper\_hengchen.pdf?attredirects = 0$

The research paper version of Chapter 2 is entitled "Mortgage Loans, the Decline of the Household Saving Rate and the Increase in Risk-Sharing " and can be downloaded from

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"Underdevelopment of Financial Markets and Excess Consumption Growth Volatility in Developing Countries "

Published: This thesis has not been published yet.

Status: It will be sent to peer-review academic journal.

Presented at/to: Meeting of the European Economic Association (24th EEA-ESEM), August 2009, Barcelona, Spain; Nordic Summer Symposium in Macroeconomics (3rd), August 2009, Smogen, Sweden; European Workshop in Macroeconomics (5th), June 2009, Mannheim, Germany; Conference in Macroeconomic Analysis and International Finance (13th), May 2009, Crete, Greece

"Mortgage Loans, the Decline of the Household Saving Rate and the Increase in Risk-Sharing "

Published: This thesis has not been published yet.

Status: It will be sent to peer-review academic journal.

Presented at/to: Conference on risk sharing, Zurich, June 2, 2010

"Why is the Correlation Between Savings and Investment So Low in Developing Countries? "

Published: This thesis has not been published yet.

Status: It will be sent to peer-review academic journal.

Acknowledgments

My Ph.D. studies have been an extraordinary experience in my life. That is an experience with hard work, painful frustration and luckily, joy of success. I was very fortunate to study in three great institutions and to work with so many gifted individuals. I would like to take this opportunity to express my deepest gratitude to those who have helped, encouraged and inspired me in the past few years.

First and foremost, I am forever indebted to my advisor Fabrizio Zilibotti for his excellent guidance. In Stockholm, I took two courses with him, that uttered me in the research fields, in which I wrote this thesis. He was extremely generous with his time and always patiently discussed with me about those even very preliminary ideas I had. In Zürich, I worked on my thesis under his supervision and benefited tremendously from his inspiring ideas, invaluable suggestions and constructive comments. With his guidance, I have learned how to formalize an research idea and transform it into a scientific paper. With his comments, I have developed a taste for good research and understood how it should be done. Not only was he always available for discussion on my research, but also he spent a great deal of time and efforts to help me acquire the writing and presentation skills that an economist needs. His passion for economics had a great impact on me and all the others around him. I feel very fortunate and privileged that I can have him as my mentor on the journey of becoming an economist.

I am also sincerely grateful to Christoph Winter, who is a resourceful co-author and great friend. It has always been a pleasure to work with him and to explore the uncharted territory altogether. I also wish to thank Zheng Song, who has been a successful role model for me and has helped me to make several right decisions that were particularly important for my studies and career. My thanks also go to Yikai Wang, whom I shared the office with in the last two years. He is one of the most intelligent friends I have. It has been a great fun to study new methods and discuss new ideas with him. I am also indebted to Marcus Hagedorn for his valuable clarification and constructive criticism at different stages of my research.

I also owe so much to those that I have worked and studied with at the three institutions of my graduate years.

I spent the first two years of my Ph.D. studies at the Stockholm University, where I have many friends that I would like to thank. I always remember the enjoyable time I spent with Jan Klingelhöfer (now at EUI) to solve problem sets and prepare for exams. I am grateful to Maria Perrotta, who helped me with the data work for the first paper I wrote. I appreciate Jose Mauricio Prado Jr. (now at Cambridge), who generously gave to me his C codes, which got me started with coding in C++. I want to express my thanks to those great Swedish friends of mine, Anna Dreber(SSE), Magnus Rödin and Maria Cheung. They had helped me in so many ways, when I lived in Stockholm. Last but not least, I am also grateful to my Chinese and Taiwanese friends in the department, Jinfeng Ge, Carol Chen (now at Taizhong), Ruixue Jia, Bei Qin and Yongan Li .

The year I spent at Harvard University was both academically productive and intellectually stimulating. I was delighted to know many talented and wonderful friends. I would like to thank Loukas Karabarbounis (now at Booth, Chicago) who constantly helped me, when we studied together, and who enthusiastically encouraged me, when we went on the job market. I am also particularly grateful to Daniel Shoag, who spent tons of time to study with me and also had invited me for wonderful Jewish dinners several times. I owe special thanks to Keyu Jin (now at LSE), Danxia Xie (now at Chicago) and Ruchir Agarwal, with whom my time at Harvard was even more enjoyable.

I had started and finished writing my thesis at the University of Zürich in the past three years. It has been the most stressful and challenging period in my life. Luckily, I got to know a group of friends, who helped me go through this process. I would like to thank, Andreas Müller, Sigrid Röhrs, Simon Alder and Philippe Sulger, who generously helped me in a variety of situations. I am also grateful to Bernhard Ganglmair and Filippo Brutti, who helped me to prepare for the job market. I am also indebted to other members of "the Chair", Michelle Rendall, Dominic Rohner, Andreas Beerli, Sebastian Findeisen, and Franziska Weiss. I am especially grateful to Stephanie Raimander for her excellent administrative assistance.

I have presented parts of this thesis at various conferences, seminars, and workshops and would like to thank the participants for their comments and discussions. Particularly, I owe so much to Paul Klein, who was the discussant for my job market paper at the 3rd Nordic Macroeconomics Workshop. He has also kindly provided so many detailed comments on that paper in follow-up correspondences.

Moreover, I would also like to thank the Handelsbankens and Widar Bagge Foundation for their generous financial support and Jonas Häckner for his efficient coordination.

I owe special thanks to my friends outside academia, especially Clara Chen, Yingying Ma and Kun Zhu, who have always been there for me during difficult times. I am deeply grateful to my bother Ping and sister-in-law Ying, who took care of the family when I was far away from home.

Finally, I would like to thank my mother for her love and support. She taught me that everything is possible with patience, perseverance and courage. Without those, my Ph.D. would not have been achieved. I dedicate this thesis to her.

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Introduction

This thesis consists of three macroeconomic essays, dealing with various issues concerning the dynamics of savings and consumption. Chapter 1 focuses on the excess consumption growth volatility in developing countries. Chapter 2 (joint with Christoph Winter) deals with the changes in saving rate, housing consumption and risk-sharing in the U.S. economy for the period 1980 - 2000. Chapter 3 studies the savings-investment correlation in low-income countries.

Chapters 1 and 3 are closely related to each other: They explain differences in macroeconomic performance between developing and developed countries. These chapters contribute to a growing literature, which aims at identifying structural differences between the two types of economies. Both of the chapters argue that the natures and effects of productivity shocks are important determinants of the distinct economic dynamics observed in developing countries.

Chapter 2 explains the aggregate trends in the U.S. economy during the era of "financial liberalization". It shows quantitatively that the institutional changes in housing mortgage markets can account for a significant share of the decline of the saving rate, as well the increase in home-ownership rate and mortgage debt.

Methodologically, all three essays are motived by stylized facts, which are properly documented either in this thesis or by previous empirical literature. I explain those statistical regularities with quantitative macroeconomic models that allow realistic features. This approach has become a mainstream tool for macroeconomics in the last 30 years. The advantage is that it allows one to assess quantitative implications of macroeconomic theories. Macroeconomists have been using model calibration to test a variety of theories and to evaluate alternative policies. Following this tradition, this thesis provides three calibrated quantitative models to evaluate to what degree the proposed theories can match those stylized facts in the data.

The three essays rely on micro-founded dynamic stochastic general equilibrium (DSGE) framework. This framework is particularly attractive for two important reasons. Firstly, micro foundations provide modern macroeconomic models with coherent structures and explicit guidelines for empirical investigation. Secondly, the framework also helps to identify policy implications of macroeconomic models, that are immune to the so-called "Lucas critique".

This thesis makes use of two variants of DSGE models: representative-agent and heterogeneous-agent models. Representative agent models have been popular in macroeconomics, as they provide relatively simple shortcuts to study behaviors of aggregate variables. Chapter 1 and Chapter 3 rely on the representative agent framework and focus on how consumption and savings evolve over time.

Chapter 2 offers a model, which is built within the heterogeneous-agent framework. Models with heterogeneous agents are usually developed to address certain features of the data, which cannot be explained within the representative agent framework, for instance, consumption inequality, wealth distribution, etc. Heterogeneity is important and necessary in this essay, since one of its focuses is risk-sharing opportunities among households, which cannot be properly addressed in a model with only a representative agent. It, therefore, explicitly deals with the stationary distribution of the households over the state space of age, productivity, debt, financial assets and housing wealth.

The models in the three essays in general do not admit closed-form solutions, unless under special parameter values. Therefore, I have to rely heavily on various numerical methods to solve and simulate these models. Broadly speaking, there are two important lines of numerical methods to solve quantitative macroeconomic models: the value function approach and the Euler-Equation approach. Both of them have been applied in this thesis. Chapter 1 and 3 present algorithms to solve functional equation systems, which are derived from first order conditions and the envelop theorem, etc. The major difference is that Chapter 1 solves for highly non-linear policy functions with the Broyden's method and Chapter 3 makes use of the linearization techniques to obtain a linear form of laws of motion for the state and choice variables around the steady state.

In Chapter 2, however, we resort to the value function iteration method . Since in that context, the Euler equation approach is hard to apply, due to the discontinuities in policy functions. Fortunately, the method of value function iteration with grid search proves to be very robust. However, it is a well-known issue that the computing time increases exponentially, when the discretization of state space increases linearly. Parallel computing techniques that we use are shown to be helpful to speed up the computation. This chapter illustrates how the cutting-edge computing technology can be applied in solving quantitative models in macroeconomics.

In what follows, I briefly summarize the content and main results of each chapter.

Chapter 1 (*Underdevelopment of Financial Market and Excess Consumption Growth Volatility in Developing Countries*) aims at explaining the excess consumption growth volatility puzzle in developing countries. It is a well-documented fact that the output growth volatility is substantially and significantly higher in developing countries (Acemoglu and Zilibotti 1997). It is not surprising to observe that the consumption growth volatility is also higher, considering the output growth volatility as underlying macroeconomic volatility. More interestingly, Kose, Prasad, and Terrones (2003) find that the negative relationship between volatility and development is even more pronounced in the case of consumption growth volatility. In other words, consumption growth volatility in developing countries is disproportionately higher than in developed countries, relative to output growth volatility.

The purpose of this essay is to construct a theory that is consistent with these observations. The focus on the excess consumption growth volatility is well justified: consumption growth volatility has immediate welfare implications than the output growth volatility. If, for instance, it were the case that poor countries can insure themselves through international risk-sharing, consumption growth can be fairly stable and the welfare costs of output fluctuation would be less significant. However, that is not the case in reality.

I build a DSGE model by following the seminal work by Acemoglu and Zilibotti (1997), which shows how the lack of diversification leads to the high output growth volatility in developing countries. I embed their endogenous diversification mechanism into a rather standard stochastic growth model. There are two important channels, with which the dynamics of consumption behaviors are enriched. Firstly, similar to the Acemoglu and Zilibotti (1997), there is an amplification channel in this model, where uncertainties in the financial markets are amplified through the capital accumulation process. Secondly and more importantly, I show that the exogenous uncertainties can be amplified by the first channel, so that the effects of TFP shocks on the future output can be more persistent.

These two mechanisms imply more volatile consumption (relative to output) in developing countries than in developed countries. First, while output just keeps track of the capital level, consumption responds even more to endogenous shocks from the financial market, since these shocks have persistent effects on future output and consumption opportunity through the amplification channel. Since the developing economy is in a less complete financial market most of the time, this effect is stronger in the developing economy. It implies that the ratio concerned should be relatively higher in developing countries.

Second, exogenous TFP shocks are amplified by endogenous shocks from the financial market. Therefore, the persistence effect on output of exogenous

TFP shocks is endogenously higher. Consumption also responds to this effect and becomes more volatile. Once more, this type of interaction plays a larger role in the developing economy. It is almost absent in the developed economy, since the financial market is complete most of the time.

Interestingly, this model shows that the consumption behavior in developing countries can be quite different from those in developed countries, given we assume that both of them are subject to the same productivity shock process.

The above conclusion has an interesting implication on identifying the shock properties in developing countries. I conduct the following experiments to show that the identification can be biased, if we ignore the underlying structure in developing countries, i.e. the endogenous incomplete financial market. Suppose that the model proposed in the essay captures the reality exactly, but that one tries to understand the data produced by the model, with a "misspecified" model, where an exogenous difference between the TFP shock processes for the two types of economy is assumed. The estimation of the artificial data concludes that there is a difference between the TFP processes of the two groups, i.e., the permanent component in the developing countries case is larger. Obviously, the observed difference should be attributed to the ignored endogenous diversification channel.

Chapter 2 (*Mortgage Loans, the Decline of the Household Saving Rate and the Increase in Risk-Sharing* (with Christoph Winter)) studies to what extent the changes in housing finance during the mortgage market deregulation can quantitatively account for the changes in aggregate trends, such as the decline of personal saving rate, increase in mortgage debt and homeownership rate.

Between 1980 and the beginning of the current century, the U.S. economy experienced a sharp decline in the personal saving rate. It is a well-known fact and one of the most important changes in the U.S. economy. Another way to confirm the shift of resources in the U.S. economy is to observe the other side of the coin– consumption as share of income, which has been increasing over the same period. Moreover, mortgage debt has undergone a dramatic increase over the interested period as well – the mortgage debt to labor income ratio has increased roughly 15 times. Meanwhile, the homeownership rate in the U.S. economy has been rising, precisely from 64 % to 69 % .

We present a quantitative life cycle model with housing and mortgage loans, which allows us to model the impact of financial deregulation. The life-cycle model consists of households, who are ex-ante identical and become heterogeneous over their life cycle, due to stochastic income shocks they are subject to. We incorporate housing into this model, where houses serve both as consumption goods and assets. More importantly, we model both the traditional mortgage loan structure and refinancing opportunities to represent a stylized contrast of the U.S. economy before and after the financial liberalization.

We argue that refinancing is the key element of the financial liberalization. Households can borrow against home equity, when adverse income shocks occur. When they can refinance, households decrease precautionary saving and increase both housing and nonhousing consumption. Moreover, when refinancing is allowed, housing wealth becomes more liquid. Therefore, households are more willing to accumulate housing stock and raise more collateralized debt as a consequence. We also find that the decrease in down-payment requirements allows homeowners to make use of even larger share of the home equity and therefore amplifies the effects of refinancing.

Consistent with the data, our model delivers a substantial increase in both net mortgage debt and homeownership rate, as well as substantial part of the increase in total consumption share.

Chapter 3 (*Why is the Correlation Between Savings and Investment So Low in Developing Countries?*) studies an interesting stylized fact, which is related to the famous Feldstein-Horioka puzzle. Feldstein and Horioka (1980) argue that the correlation between savings and investment should be quite low in a world with well integrated financial markets. However, they show that even for the developed OECD countries, domestic savings and investment co-move quite closely.

I focus on a related issue, which has not attracted as much attention as the Feldstein-Horioka puzzle. As shown in various studies, both *cross-section wise* and *time series wise*, not only the savings-investment correlations are high, but also they are even higher for industrial countries than for the developing countries. Existing theories with "frictional market approach" seem to be at odds with this empirical pattern: If developing countries are less integrated into international markets, it would imply a higher correlation between savings and investment.

This chapter shows that the savings-investment correlation is affected by the relative importance of the permanent (or transitory) component in the productivity shock process. Permanent income theory suggests that both savings and investment move in the same direction, given a small open economy hit by a transitory shock. Or, in other words, transitory shocks trigger a positive co-movements between savings and investment, which results in a positive correlation between these two variables. In contrast, savings and investment respond to the permanent shock in different directions, which results in a negative correlation. If the permanent (transitory) shocks account for larger component the shock process, the correlation will be relatively lower (higher).

Empirically, one of the remarkable and fundamental features of lower income economies is that non-stationary shocks (or shocks to trend growth) are the primary sources of their fluctuations. In contrast, stationary shocks (or transitory fluctuations around a stable trend) are driving the business cycles for industrial economies (Aguiar and Gopinath 2007). If a developing economy is hit by a productivity shock, it is more likely to be a trend shock, then savings and investment will be less positively correlated. In contrast, if a developed economy is hit by a productivity shock, it is more likely to be transitory, then savings and investment will be more positively correlated. Therefore, the low savings-investment correlation in developing countries emerges as the consequence of their underlying shock properties, which are different from developed countries.

Chapter 1

Underdevelopment of Financial Markets and Excess Consumption Growth Volatility in Developing Countries

Chapter Summary

This paper aims at explaining, both qualitatively and quantitatively, why consumption growth is substantially more volatile in developing countries than in developed countries. I propose an infinite-horizon stochastic growth model with endogenous financial development, à la Acemoglu and Zilibotti (1997). In this model, micro-level project indivisibility and aggregate savings determine the degree of diversification in financial markets. In addition, countries are subject to TFP shocks with different means, capturing differences in technology, but with equal variance and persistence. On average, less technologically advanced economies have lower income and savings, translating into lower financial development. When the financial market is underdeveloped, shocks to investments and TFP endogenously have more persistent effect on future output. Thus, consumption responds more to those shocks, and the volatility of consumption relative to the volatility of output is higher in poorer than in richer countries. I also show that a calibrated version of the model is consistent with a number of features of the data, without relying on exogenous differences in the variance and persistence of TFP shocks.

Key words: Financial Market, Diversification, Consumption Growth Volatility

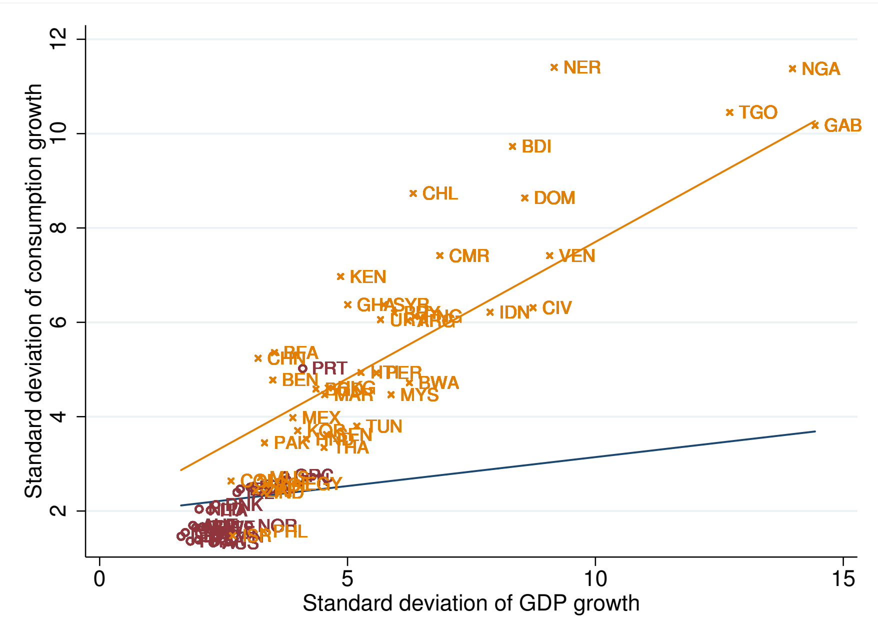
JEL classification: E2,O1

1.1 Introduction

This paper aims at explaining the excess consumption growth volatility puzzle in developing countries. The data suggest that output growth is generally more volatile in those countries. More interestingly and puzzlingly, the negative relationship between volatility and development is even more pronounced in the case of consumption growth volatility. In other words, consumption growth volatility in developing countries is disproportionately higher than in developed countries, relative to output growth volatility (Kose, Prasad, and Terrones 2003). The purpose of this paper is to construct a theory that is consistent with these observations.

The focus on consumption growth volatility is well justified. The extent to which high volatility is a first-order problem for developing countries depends on the extent to which output growth volatility translates into consumption growth volatility. If, for instance, it were the case that poor countries can insure themselves through international risk-sharing, consumption growth can be fairly stable and the welfare costs of output fluctuation would be less significant. However, that is not the case in reality. Evidence shows (e.g. Lewis 1996) that international consumption risk sharing is quite limited. This implies that reducing volatility in developing countries would potentially entail substantial welfare gains.

Figure 1.1: Output and Consumption Growth Volatilities



Source: WDI data, 1960-2007. Regression of the standard deviation of consumption per capita growth on the standard deviation of GDP per capita growth.

Table 1.1: Excess Consumption Growth Volatility

	σ_c	σ_y	σ_c/σ_y
Developed countries	2.155	2.403	0.896
	(0.46)	(0.31)	(0.07)
Developing countries	5.385	4.503	1.197
	(0.34)	(0.19)	(0.05)
Difference	3.23	2.101	0.302
	(0.57)	(0.36)	(0.08)

Source: WDI (1960 - 2007). All the numbers are reported in percentage. σ_c and σ_y are standard deviation for consumption growth and output growth, respectively. σ_c/σ_y is their ratio. Standard errors are in parenthesis.

Using WDI data from 1960 to 2007, I regress the standard deviation of consumption per capita growth on the standard deviation of GDP per capita growth and country group dummy.[1] Figure 1.1 shows the regression lines for developed and developing countries, respectively. Developed countries cluster around the lower left corner, which means that both consumption and GDP growth volatilities are low. The picture for developing countries is quite different: Most of them spread out towards the upper right corner, which means that both volatilities are higher in developing countries. Moreover, consumption growth seems to present excess volatility: Consumption growth volatility increases much more in response to GDP growth volatility. The positive slope of the regression line for developing countries is significantly higher.

To more clearly identify this pattern, I analyze the ratio of consumption per capita growth volatility to GDP capita growth volatility. Table 1.1 gives the average standard deviations of consumption and GDP growth as well as their ratios in developing and developed countries, respectively. In the second column, the negative relationship between output growth volatility and income level is obvious, while the first column shows that the same relationship also holds for consumption growth. The third column gives the mean ratios in each group and shows that the average ratio is disproportionately higher in developing countries. The gap between the two averages, roughly 0.3, is large and statistically significant.

Similar exercises have been conducted using different data in terms of sample

[1] More than twenty industrial economies are refereed to as developed countries and the remaining countries of the sample, which have a lower income level, are labeled as developing countries. Note that very small countries, countries with clearly unreliable data and oil producers are excluded from the analysis. Consumption and GDP are both in real per capita terms and in constant local currency unit.

countries, time interval and frequency.[2] Kose, Prasad, and Terrones (2003) document a similar pattern, although the gap that they find is relatively smaller than mine.[3]

The existing literature tries to explain why consumption is substantially more volatile in lower income countries by relying either on different properties of exogenous shocks (e.g. Aguiar and Gopinath 2007) or on international channels (e.g. Levchenko 2005, Resende 2006 and Neumeyer and Perri 2005). In contrast, this paper shows that the frictions in domestic financial markets can help explain the empirical puzzle.

I propose an otherwise standard stochastic growth model with an infinite horizon. The new element is that the financial market is explicitly modeled, à la Acemoglu and Zilibotti (1997). I assume that agents have access to a large number of imperfectly correlated risky projects in the intermediate sector which transform savings into capital goods, only one of which succeeds each periods, i.e. delivers a positive return. Those projects receive savings from households by issuing securities in the financial market. When the uncertainty has unraveled, the productive project distributes output to security holders. In addition, some of the risky projects are required to raise a certain amount of savings from individuals, before being productive. If there are not enough savings in the economy, some projects are not funded and thus, not all securities are available in the financial market. In contrast, if there are enough savings in the economy, minimum size requirements are irrelevant and the financial market is complete.

If the financial market is incomplete, the return to investment in risky securities is stochastic. It is a good (bad) draw if the productive project is (not) funded and yields (does not yield) a return. Good draws result in more capital goods being brought forward to the next period. It implies higher savings, which helps to better diversify the risks in the intermediate sector. This, in turn, increases the chances of receiving good draws in the following periods, hence increasing expected future income.[4] In other words, shocks in the financial market *amplify* themselves through capital accumulation.

[2] Aguiar and Gopinath (2007) also lend support to this finding with a relatively small sample of emerging and industrial economies. Their data suggest that emerging economies exhibit relatively volatile consumption at business-cycle frequencies, even though the already high income volatility is controlled for. Resende (2006) studies a sample of 41 small open economies. His findings are well consistent with previous research. Similarly, De Ferranti et al. (2000) show that the volatility of the growth rate of real GDP in Latin American countries is twice as high as in industrial economies, while consumption growth volatility is three times higher than in industrial economies.

[3] They construct an income measure based on GNP. The standard deviation of income growth is higher than that of output growth in both types of economy. They report the gap between the within-group medians.

[4] Similarly, bad draws do not only decrease the capital stock, but also reduce the probability of good draws in the following periods, and further reduce the future expected income.

Unlike Acemoglu and Zilibotti (1997), this model allows exogenous TFP shocks and their *interaction* with endogenous shocks from the financial market. I assume countries to be subject to TFP shocks, which have exactly the same variance and persistence. The only exogenous difference between developing and developed economies is the long-run mean TFP, which captures the difference in technological or productivity levels.

Importantly, the difference in development levels translates into the difference in diversification of the financial market. In this model, the developed economy behaves similarly to a standard stochastic growth model. Its steady-state level of capital is sufficiently high to afford a fully diversified financial market and all idiosyncratic risks are diversified. Most of the time, it is fluctuating around the (deterministic) steady state, with the complete financial market. The interesting difference as compared to standard stochastic growth models is that a sequence of bad TFP shocks could shift the fully diversified economy away from the steady state and back to the situation where the financial market is less complete and the economy could be hit by bad draws. Thus, the model predicts both frequent "small recessions" and rare deep and persistent recessions in developed countries.

On the other hand, since the developing economy is less productive and the "steady-state" level of capital is so low, the fully diversified financial market is not affordable. The economy is always subject to shocks to investment from the intermediate sector, so that the volatility in both consumption and output will be higher than in the developed economy. The model also predicts that the output gains during expansion can be larger in lower income countries. To understand this, suppose that the economy is hit by a sequence of good TFP shocks. They lead to higher savings and allow the economy to expand, which improves the diversification opportunities in the financial market. This, in turn, implies a higher chance of getting good draws from the financial market. Booms are reinforced and stronger. This prediction is well consistent with the empirical findings in Calderón and Fuentes. (2006).[5]

The two important mechanisms (amplification and interaction) imply more volatile consumption (relative to output) in developing countries than in developed countries. First, while output just keeps track of the capital level, consumption responds even more to endogenous shocks from the financial market, since these shocks have persistent effects on future output and consumption opportunity through the amplification channel. Since the developing economy is in a less complete financial market most of the time, this effect is stronger in the developing economy. It implies that the ratio concerned should be relatively higher in developing countries.

Second, exogenous TFP shocks are amplified by endogenous shocks from the financial market. Therefore, the persistence effect on output of exogenous TFP

[5]They show that expansions are, on average, stronger in lower income countries (e.g. Asian developing and Latin-Americans countries) than in industrial ones. In particular, they show that Colombia and Malaysia achieved the largest output accumulation during the expansion phases.

shocks is endogenously higher. Consumption also responds to this effect and becomes more volatile. Once more, this type of interaction plays a larger role in the developing economy. It is almost absent in the developed economy, since the financial market is complete most of the time.

This paper also sheds some light on the link between frictions in the financial market and observed differences between *measured* TFP shock processes (e.g. Aguiar and Gopinath (2007) hypothesize that the TFP shock properties are different across groups.)[6]. I assume there to be no exogenous difference in shock processes between the two types of economy. Instead, I study how a standard stochastic growth model, enhanced by the friction of micro-level project indivisibility, could endogenously deliver the observed differences between *measured* TFP shock processes.

The quantitative results show that the model can replicate the empirical pattern pretty well. The ratio of consumption growth volatility to output growth volatility is substantially higher in the developing economy case. The gap generated by the model accounts for a substantial part of the data. The model also predicts that an increase in the technological level is associated with a decrease in both consumption and output growth volatilities. Moreover, consumption growth volatility should drop even more quickly. The results from simulation data also confirm this prediction.

The paper relies on the endogenous diversification channel, proposed by Acemoglu and Zilibotti (1997). Apart from the fact that the main focus is the consumption volatility puzzle, there are noteworthy differences between this model and their work. First, I model an economy with an infinite horizon which is better suited for studying high-frequency phenomena, in contrast to the two-period OLG framework in their paper, which is appropriate for development issues. Second, exogenous TFP shocks are included, so that it is possible to quantitatively assess the model economy with the data. More importantly, the interaction between endogenous and exogenous shocks arises in this model. Third, I impose more general assumptions on preferences and depreciation, which yield important new insights and turn out to be critical for solving the puzzle.[7] Finally, the general setup of this model poses technical challenges. The numerical analysis of the paper provides a functional and successful algorithm for solving the general framework.

This paper finds its place in the growing literature on consumption volatility. One group of research relies on the international sector to address the question of why increasing international financial integration is associated with higher consumption volatility in more financially integrated developing countries.

[6] Measured TFP processes is constructed from Solow residual, where the capital stock is measured as the sum of past investment, assuming that one unit of saving translates into one unit of investment in a closed economy.

[7] They assume logarithm utility and full capital depreciation in their model, which allows them to derive analytical solutions. However, the simplicity comes at a cost: Substitution effect, income effect and wealth effect cancel out exactly. The savings rate is constant and therefore, the relationship between consumption growth volatility and output growth volatility cannot be properly studied.

For example, Resende (2006) hypothesizes that developing countries are borrowing constrained and therefore, the lack of ability to smooth their consumption renders the ratio higher than in developed countries. He finds that this mechanism alone has a rather limited explanatory power.[8] Neumeyer and Perri (2005) propose that shocks to the country risk premium could provide another source of uncertainty and also amplify the exogenous TFP shocks, if the default risk premium is negatively correlated with TFP shocks. They claim that through this channel, consumption can be more volatile than output in emerging economies. Levchenko (2005) adopts the Kocherlakota (1996) framework of risk sharing subject to limited commitment to explain why consumption volatility can be higher, if lower income countries are better integrated into the international market.

While this line of research is successful to different degrees, no explanation is offered as to why the relative consumption growth volatility differential still exists in less financially integrated developing countries. This empirical fact can be readily explained by this model.

Another line of research focuses on the different properties of TFP shocks in emerging countries. Aguiar and Gopinath (2007) argue that industrial and lower income countries undergo different underlying income processes. Their hypothesis is that there are two components in the productivity shock process, transitory and permanent. In industrial economies, the transitory shocks are relatively more important, while in poorer emerging economies, the permanent component plays a larger role. Their theory implies that consumption is relatively more volatile in lower income countries.

Although they also point out that the difference in TFP processes might be a manifestation of deeper frictions in the financial market, they do not focus on how the financial frictions translate into the observed differences in TFP processes. This paper attempts to provide a link between these two.

This paper is also related to research focusing on the relationship between diversification and macroeconomic volatility (Acemoglu and Zilibotti 1997, Imbs and Wacziarg 2003, Koren and Tenreyro 2007a and 2007b, and Kalemli-Ozcan et al. 2009). In contrast to previous research which puts emphasis on output growth volatility, this paper tries to explain the consumption volatility pattern. It also stresses the importance of the interaction between aggregate shocks and the diversification channel, which is absent in the previous literature.

The rest of the paper is organized as follows. The next section presents the basic model and characterizes the equilibrium. A numerical example is used to explain the basic mechanisms in the model. Section 3 explains the calibration and simulation strategy and Section 4 presents the basic findings. The empirical pattern found in the data is compared with the numerical results. The model is shown to be consistent with a number of features of the data, without relying on any exogenous differences in the variance and persistence of the TFP shock process. Section 5 concludes the paper.

[8]He suggests that the reason why consumption volatility cannot exceed income volatility is due to the lack of permanent shocks in his model.

1.2 The Model

1.2.1 Environment

The decentralized model economy is populated by infinitely lived agents. A constant relative risk aversion utility function is assumed to parameterize their preferences. Agents maximize their expected life time utility, which is defined by

$$U = E_0 \sum_{t=1}^{\infty} \beta^{t-1} \frac{c_t^{1-\sigma}}{1-\sigma}$$

where c_t is consumption in period t, σ is the coefficient of relative risk aversion and β is the discount factor. The population is constant and normalized to be one. Labor supply is assumed to be inelastic and, therefore, it is also constant.

The production side consists of two sectors, the final good sector and the intermediate sector. The final good sector uses capital and labor to produce a final output. The production function in the final good sector is assumed to be Cobb-Douglas with capital K_t and labor L_t as inputs

$$Y_t = A_t K_t^{\eta} L_t^{1-\eta}$$

where $\eta \in (0,1)$ is the elasticity of output to capital and A_t is productivity in period t. Productivity is subject to an aggregate shock.[9] Formally, $A_t = e^{z_{it}}$and z_{it} follows an $AR(1)$ process

$$z_{it} = (1-\rho)\,\mu_i + \rho z_{it-1} + \varepsilon_t$$

where $|\rho| < 1$ and ε_t is a serially uncorrected normally distributed random variable with zero mean and constant variance, that is $\varepsilon_t \sim \mathcal{N}(0, \sigma_z)$. μ_i is a constant and i is the country type dummy: 0 stands for developing countries and 1 for developed countries. It characterizes the difference between developing and developed countries: $\mu_0 < \mu_1$.[10]

[9]Note that the growth trend shock is an important source of volatility in output and consumption growth in developing countries, which has been studied by Aguiar and Gopinath (2007). Since my goal is to explore and highlight the underdevelopment of financial markets and its effects on consumption growth volatility, I assume away the growth trend of productivity or, in other words, assume the exogenous productivity growth to be zero. This can be considered as a de-trended version of a more general model. I provide a version of this model with a deterministic trend in the Appendix and show that it is not essential for the results.

[10]Note that it is the only exogenous difference I assume between these two groups. In a more general setup, I could assume there to be a stochastic type-switching process: Each type of economy has some probability of switching to the other type, governed by an exogenous transition matrix. The switching probability is usually quite low. To keep the model simple and the results sharp, I assume that the switching probability is zero.

Agents work in the final goods sector and earn a competitive wage and also receive capital income through the competitive renting market. Prices, precisely wage rate and return to capital, are competitively determined by aggregate capital in the economy, K_t, and the productivity level, A_t. Agents decide how much to consume and save every period. They are also allowed to decide on the allocation of their savings in the financial market.

Following Acemoglu and Zilibotti (1997), I assume there to be an intermediate sector, which transforms savings into capital goods brought forward to the next period without using any labor. There is uncertainty which is represented by a continuum of equally likely states $state \in [0, 1]$. The transformation technology takes two forms: Safe and risky projects. The safe project gives the non-stochastic return r. There is a continuum of risky projects, corresponding to the states of nature. Risky project j pays a positive return only in state $j \in [0, 1]$ and zero otherwise.

Risky projects are financed by issuing securities in the financial market. Output from the risky projects is entirely distributed to the holders of securities. No profit is retained. The payoff to security holders in state of nature j is $R \cdot F_j$, if security holders invest F_j (density) in security indexed by j. It is assumed that $R > r$, which is consistent with the intuition that risky assets give a higher return. Note that not all the projects are necessarily funded, and therefore not all the securities are available in the economy. The measure of available securities, n_t, is determined in equilibrium.

In addition to deciding on savings (and consumption) in each period, agents are also allowed to decide how they allocate their savings in the financial market, i.e. the portfolio decision. They can invest in a set of available risky securities ($i \in [0, n_t]$), which consists of state-contingent claims to the output of the risky projects, and the safe asset, which consists of claims to the output of a safe technology. The assets portfolio is defined by α, which is the percentage of savings invested in the safe asset. It is assumed that $\alpha \in [0, 1]$, which means that the agent is not allowed to borrow to invest in risky or safe assets.

The agents invest an equal amount of savings in risky securities, F, due to the symmetry of risky assets: The expected return to each risky security is exactly the same. Moreover, they would invest in *all* available securities, so that the variance in the payoff from risky investment is minimized, while the expected return is the same. That is, $F_j = F_i = F$, $\forall i, j \in [0, n_t]$. This is called "balanced portfolio" . [11]

In this model, only one type of friction is introduced, namely micro-level project indivisibility or minimum requirement of investment: The project, indexed by $j \in [0, 1]$, is productive only if it attracts at least a minimum amount of savings from individuals (see Figure 1.2), $M(j)$. One example is railway production: Building

[11] It can be shown the expected return r_E is constant. $r_E = F \cdot n \cdot R + (1 - n) \cdot F \cdot 0 = (1 - \alpha) \cdot s \cdot R$, which is the same, independent of n. The variance is decreasing in n, the measure of risky securities in which agents choose to invest, $Var = [(1 - \alpha) \cdot s \cdot R]^2 \left[\left(1 - \frac{2}{R}\right) + \frac{1}{R^2 n}\right]$.

a railway requires a large amount of investment before the project becomes useful and productive. To capture the heterogeneity in minimum size requirement across projects, it is normalized to zero for projects $j < \gamma$, while the minimum size of the rest is linearly increasing in their index.[12] Formally, the minimum size is specified by

$$M(j) = max\left\{0, \frac{D}{1-\gamma}(j-\gamma)\right\}$$

where D is the highest minimum requirement in the economy.

To appreciate the importance of this friction, consider the following case where $D = 0$ or $\gamma = 1$. Given this assumption, the micro-level project indivisibility is absent and all projects will be funded. Agents would invest an equal amount in all risky securities. The return to this portfolio becomes deterministic. Intuitively, with the assumption that $D > 0$ and $\gamma < 1$, it is not necessarily the case that all projects could attract enough savings to meet their minimum requirements.

Aggregate savings and associated portfolio choice, together with micro-level project indivisibility, help determining the measure of open projects in equilibrium. Intuitively, if savings in the economy are less sufficient, agents would invest in the safe asset to seek insurance and invest even less in risky securities. Based on the balanced portfolio, each open project would raise the same amount of savings to fund its production in the intermediate sector. In equilibrium, given the aggregate amount of savings allocated to risky projects, the maximum possible measure of projects will be less than one in the economy.[13] Suppose, on the other hand, that the savings in the economy are sufficiently high and all projects can raise enough savings to overcome the minimum requirement. The maximum possible measure of risky securities is one and the market is complete.

1.2.2 Recursive Formulation: Decentralized Equilibrium

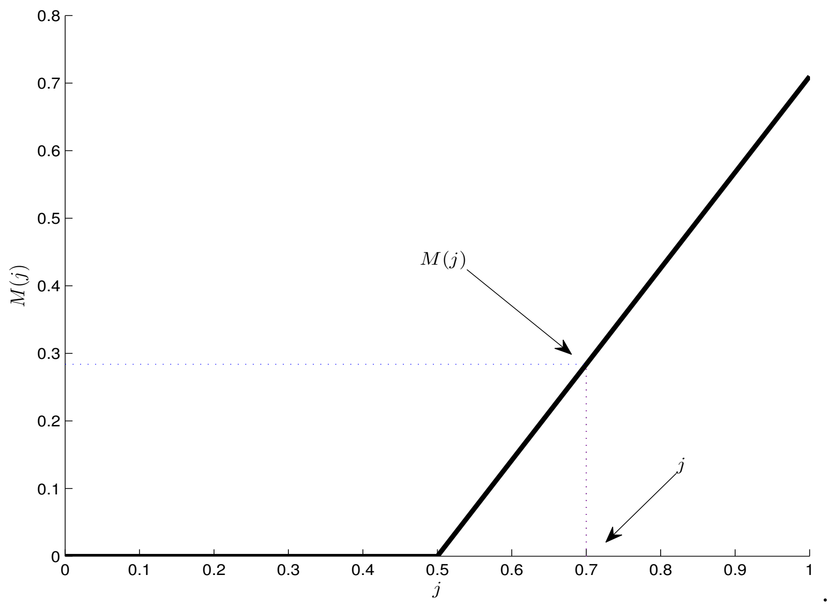
Formally, the problem solved by the representative agent can be restated in the following recursive formulation. The measure of available securities, $n(K, A)$, is a function of aggregate variables. The agent takes this as given, and solves the following problem:

$$V(K, k, A) = \max_{s \geq 0, 1 \geq \alpha \geq 0} \left\{u(c) + \beta \cdot E_{K,A} V\left(K', k', A'\right)\right\}$$

[12]The results are not driven by the specification of the linear form. Parameters γ and D will also be calibrated.

[13]Acemoglu and Zilibotti (1997) have an interesting micro foundation for justifying the mapping from aggregate resources to the maximum measure of securities. A similar mechanism applies in this model. To avoid a repetition of their analysis, I skip the static equilibrium determination in the financial market and focus on the dynamic aspect of the model.

Figure 1.2: Minimum Requirement of Investment



Note: The case where minimum investment is assumed.

The value function of the representative agent is a function of aggregate capital,[14] K, his own capital k, and aggregate productivity, A. The right-hand side of the Bellman equation consists of utility derived from current consumption and the discounted expected continuation value. The expectation is conditional on both A and K. First, the agent needs information about A to compute the distribution of A' in the next period. Second, the agent also needs to know the probability of good draws at the end of the period, since there are two possible realizations. The probability is computed using both aggregate variables, K and A, and $n(K, A)$. In other words, the expected continuation value must be conditional on K. It reflects the additional source of uncertainty in the economy, namely the endogenous shocks from the financial market.

The representative agent chooses saving and portfolio optimally. The representative agent's choice is subject to the budget constraint

[14] Aggregate capital information is important for the agent to solve the problem. First, in the decentralized economy, the agent acts as a perfectly competitive price taker, and factor prices are pinned down by aggregate variables. In a central planner version of this model, the agent's portfolio decision is different. The central planner trades off between opening more projects to diversify risks and a higher expected return. In the end, available aggregate resources help to determine the measure of active risky projects. Second, the measure of available securities, n, is necessary information for her to solve for decision rules. It is jointly determined by aggregate variables A and K.

$$c + s = w(K, A) + \varphi(K, A) \cdot k$$

where $w(K, A)$ is the wage rate, $\varphi(K, A)$ is the return to capital and k is his own capital. The representative agent takes factor prices as given and makes the savings decision, s, and thus the consumption decision, c.

The total amount invested in the safe asset is ϕ,

$$\phi = \alpha \cdot s$$

The total amount of investment in risky securities is $(1 - \alpha) \cdot s$. Recall the "balanced portfolio" : 1) The agent invests in each risky security with F and 2) the measure of securities, in which she invests, is the measure of available ones,$n(K, A)$. Therefore, the following relationship holds

$$n \cdot F = (1 - \alpha) \cdot s$$

The following discussion describes the law of motion for the three state variables. The individual capital accumulation function takes two forms, depending on the realization of state of nature (see Figure 1.3). Suppose that the state of nature j is realized at the end of the period. If $j < n$, project j is both funded and productive. The agent must have invested in risky securities indexed by j (once more, recall the balanced portfolio). The agent collects returns from both safe and risky assets. In this case, the capital in the next period, k', consists of three components: Return from safe asset, $r \cdot \alpha \cdot s$, return from risky asset, $R \cdot \frac{(1-\alpha)}{n} \cdot s$, and undepreciated capital $(1 - \delta)k$, where δ is the depreciation rate in the economy. In this case, I denote k' as k^g. Conversely, if $j > n$, i.e. project j is not funded, the agent's risky portfolio gives no return. Capital at the end of the period only consists of return from the safe asset and undepreciated capital. Similarly, in this case I denote k' as k^b.

Since all states of nature have equal chances of being realized, the measure of available risky securities, n, is also the probability for the agent of receiving a "good draw" , or $k' = k^g$. The probability of a "bad draw" is therefore $(1 - n)$ (see Figure 1.3).

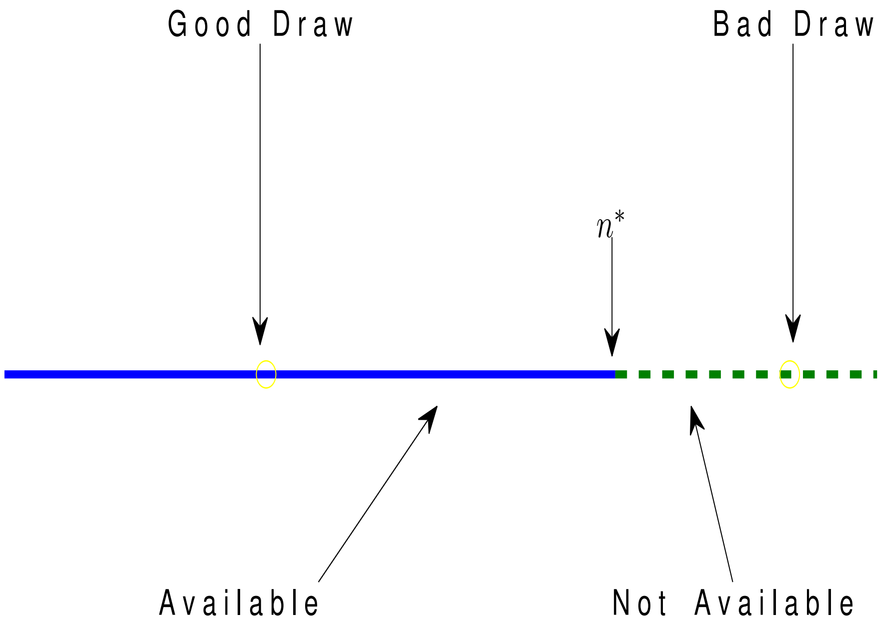
The individual capital accumulation function is therefore as follows,

$$k' = \begin{cases} r \cdot \alpha \cdot s + (1 - \delta) \cdot k & \text{if } j > n \text{ with prob } 1 - n \\ r \cdot \alpha \cdot s + R \cdot \frac{(1-\alpha)}{n} \cdot s + (1 - \delta) \cdot k & \text{if } j \leq n \text{ with prob } n \end{cases}$$

The law of motion of aggregate capital is needed for the agents to optimize.[15]

[15] The agent could choose an arbitrary belief in the law of motion of aggregate capital. In equilibrium, it must satisfy the "consistency condition" . See the equilibrium definitions.

Figure 1.3: Good Draws and Bad Draws



Note: The probability of good draws and the availability of risky securities. The solid line shows the measure of available risky securities and the probability of good draws. The dashed line shows unavailable risky securities and the probability of bad draws.

$$K' = \Psi(K, A)$$

Finally, the exogenous shock process is AR(1) [16],

$$\log A' = (1-\rho)\,\mu_i + \rho \log A + \varepsilon$$

Given the model described above, the definition of a competitive equilibrium is stated as follows:

1. $V^*(K,k,A), \alpha^*(K,k,A)$ and $s^*(K,k,A)$ solve the individual's maximization problem, taking $n^*(K,A)$ as given.
2. Prices, namely wage rate, $w^*(K,A)$ and capital return, $\varphi^*(K,A)$, are both competitively determined.
3. Consistency conditions: The law of motion of aggregate capital is consistent with the aggregation of individual capital, $K' = \Psi(K,A) = \int k'\, di$.

[16] I drop subscript i on A_i, when there is no confusion.

4. Financial market equilibrium: Given $n^*(K, A)$, the associated $\alpha^*(K, k, A)$, $s^*(K, k, A)$ and the implied $F^*(K, k, A) = \frac{(1-\alpha^*)}{n^*} \cdot s^*$, the following conditions hold:

$$F^* = \frac{D}{1-\gamma}(n^* - \gamma) \text{ if and only if } 0 < n^* < 1$$

$$F^* \geq D \text{ if and only if } n^* = 1$$

5. $K = k$.

The consistency conditions need to be elaborated. The agent knows the law of motion for the aggregate shock. She also needs to conjecture the law of motion of aggregate capital to make her decision. The conjecture, $\Psi(K, A)$, turns out to be correct and equal to the aggregation of individual capital in equilibrium.

The equilibrium conditions reflect the fact that aggregate resources and micro-level project indivisibilities jointly determine the measure of available securities. Agent $i^{'}$s investment in the available security indexed by j, $F(i, j)$, depends on his savings, portfolio choice and the availability of risky securities. That is, $F(i, j) = \frac{(1-\alpha)}{n} \cdot s$. Therefore, $\int_0^n F(i, j)\, dj$ gives the total amount of risky investment by agent i. The aggregate risky investment in the whole economy is $\int_0^1 \left(\int_0^n F(i, j)\, dj\right) di$. The equilibrium is a mapping from aggregate resources to the *possible maximum* measure of available securities, and the following condition holds in equilibrium:

$$\int_0^1 \left(\int_0^{n^*} F^*(i, j) dj\right) di \geq \int_0^{n^*} \frac{D}{1-\gamma}(n^* - \gamma)\, dj$$

where the backward inequality holds, if $n^* = 1$, and the equality holds, if $n^* < 1$. The equilibrium conditions are derived using a balanced portfolio rule.

Finally, the economy always remains on the equilibrium path. Therefore, only the case where $K = k$ is of interest.[17]

1.2.3 Optimization

Taking $n(K, A)$ as given, the agent solves the optimization problem, which reduces to two Euler equations (see the Appendix for a detailed solution),

$$U'(c) = \beta \cdot E\left[U'(c_g) \cdot R \cdot \left(\eta \cdot A' \cdot K^{g(\eta-1)} + (1-\delta) \cdot \frac{1}{r}\right)\right] \tag{1.1}$$

[17] k and K need to be distinguished when posing the decision problems of the household and firms. The equilibrium that $K = k$ is imposed *after* firms and the agent has optimized. I only need to solve for decision rules on the equilibrium path and ignore information outside the equilibrium path

$$U'(c) \geq \beta \cdot E\left[U'(c_b) \cdot \frac{(1-n)}{\left(\frac{1}{r}-\frac{n}{R}\right)} \cdot \left(\eta \cdot A' \cdot K^{b(\eta-1)} + (1-\delta) \cdot \frac{1}{r}\right)\right] \tag{1.2}$$

where $c_g\left(K^g, k^g, A'\right)$ and $c_b\left(K^b, k^b, A'\right)$ are consumption choices, given the capital stock k^g, k^b and the aggregate capital level K^g, K^b in the next period, while n is the probability of good draws,[18] given the state (K, A). The equality holds in equation (1.2), if and only if $n < 1$ and the inequality is strict, if and only if $n = 1$. The two equations are the Euler equations relating current and future marginal utilities.

In this model, the diversification opportunity is endogenous. There are two important cases. First, given a certain state, $(\widehat{K}, \widehat{A})$, $n(\widehat{K}, \widehat{A})$ can be equal to one and the backward inequality holds in the second equation. Only the first Euler equation is relevant.[19] In this case, all risks from the intermediate sector can be diversified.[20] The model behaves similarly to a standard stochastic growth model: Only exogenous shocks provide uncertainty. Second, in the other states, $n(\widehat{K}, \widehat{A})$ can be less than one and equality holds in the second equation. Both equations are relevant. In this case, the economy is subject to shocks arising from the financial market.

After imposing the equilibrium conditions, the solution is a combination of $s^*(k, A)$ and $\alpha^*(k, A)$, which satisfies the two functional equations and $n^*(k, A)$, which guarantees the financial market equilibrium condition.

1.2.4 Analytical Special Case

In general, the model has no closed-form solution. Interestingly, there is one special case, which can be solved by paper and pencil and it is the case where the analytical solution is also obtained in standard stochastic growth models. I study the special case where $\delta = 1$ and $U(c) = \log(c)$. I use the method of "guess and verify" to solve this case (see the Appendix for the detailed solution). Decision rules are solved for:

$$\alpha^*(k, A) = \frac{R \cdot (1 - n^*(k, A))}{R - r \cdot n^*(k, A)}$$

[18]The agent uses the given function $n(K, A)$ to compute the probability n at a certain state. Note that the agent can compute the decision rules, even though she takes an "incorrect " belief of $n(K, A)$. The financial market equilibrium condition is violated in that case. In other words, in equilibrium, the agent will hold a correct belief of $n(K, A)$.

[19]Given this state, the probability of a bad draw is 0. Moreover, $\alpha\left(\widehat{K}, \widehat{A}\right) = 0$ and only the savings decision, s, is of importance for the agent.

[20]If I further assume $R = 1$ and $r = 1$, the model exactly simplifies to the Euler equation in the standard stochastic growth model.

$$s^*(k, A) = \beta \cdot \eta \cdot A \cdot k^{\alpha}$$

Using the equilibrium condition, the measure of available risky securities is also solved for:

$$n^*(k, A) = \frac{(R + \gamma \cdot r) - \sqrt{(R + \gamma \cdot r)^2 - 4 \cdot r \left(\frac{(R-r)\cdot(1-\gamma)}{D} \cdot s^* + \gamma \cdot R\right)}}{2 \cdot r}$$

A few comments are in order. In this special case, the saving rate is constant, which means that exogenous shocks and capital levels do not affect the agent's saving rate. The result is the same as in the standard stochastic growth model with these two special assumptions. The portfolio choice and the equilibrium measure of risky securities are the same as in Acemoglu and Zilibotti (1997), which means that their two-period OLG model is a special case of this general framework.

The implications of this solution are that 1) the consumption rate is constant over time and it is the same in both developing and developed countries; 2) the ratio of consumption growth volatility to output growth volatility will be one, in both types of economy. The full depreciation assumption severs one crucial channel of persistence. Moreover, the substitution and income effects of intertemporal prices cancel out under log preference. In this case, the relationship between consumption and output volatility will be trivial, due to the unrealistic assumptions. This is one of the reasons why I must analyze the general model. On the other hand, this analytical solution provides a good starting point for the numerical computation of the general model, where the rooting finding method must be used and requires an exact initial guess (see the Appendix for a detailed solution).

1.2.5 Analysis

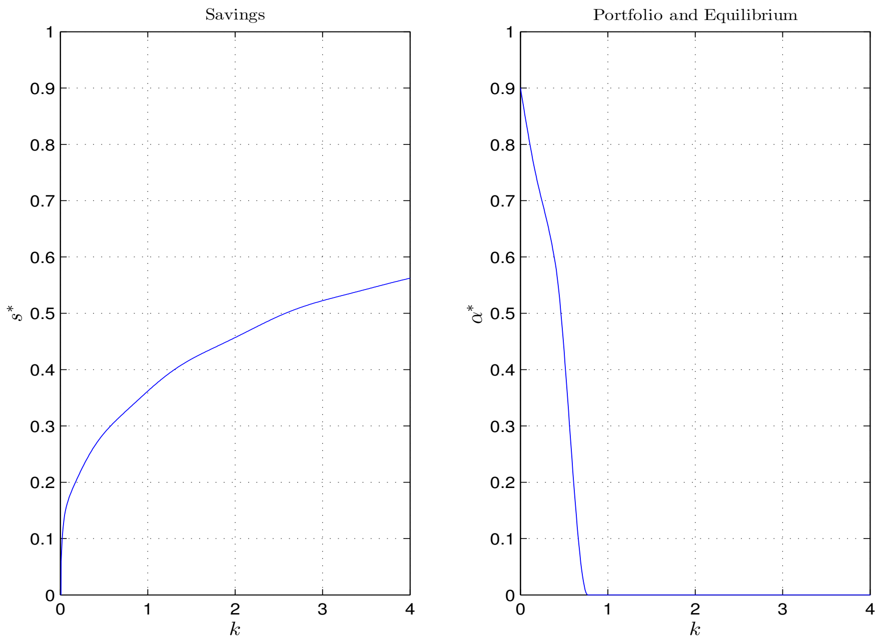
Decision Rules and Equilibrium

Policy functions, specifically the savings decision and the portfolio choice, are both two-dimensional functions of capital, k and productivity, A. The same applies for the equilibrium measure of available securities or the probability of good draws, $n(k, A)$. I present decision rules and equilibrium from a special numerical case,[21] where the exogenous shock is absent, namely, $\rho = 0$ and $\sigma_z = 0$. In this special case, the difference between the two economies is the long-run productivity levels, A_i, where A_0 stands for the developing economy case and A_1 for developed economy. The solutions are denoted by $n^*(k; A_i)$, $\alpha^*(k; A_i)$ and $s^*(k; A_i)$.

[21] See the next section for the solution algorithm.

The decision rules are similar in both economies. I plot the decision rules for the developing economy case with A_0 in Figure 1.4 and the developed economy case with A_1 in 1.5. The left-hand plot gives the savings decision, which is concave and increases in capital. The decreasing curve in the right-hand plot is the portfolio decision, α. For a given productivity level, the higher is the capital, the less do agents invest in the safe asset. Up to a certain point, the investment in the safe asset is positive. After the threshold, agents invest nothing in the safe asset. The intuition is that agents with low capital would invest in the safe asset to seek insurance. If agents are sufficiently rich, they invest all their savings in risky securities to seek a higher return.

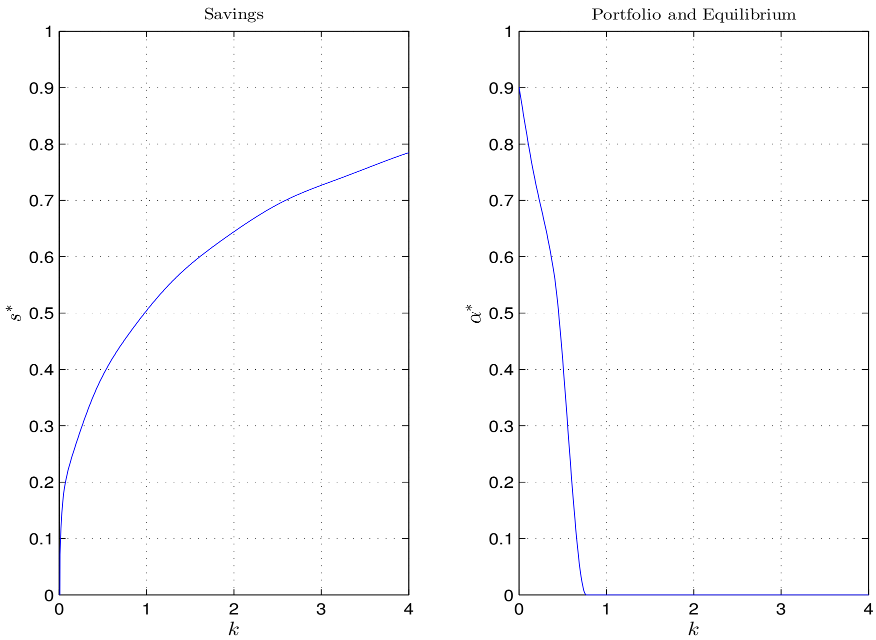
Figure 1.4: Decision Rules with A_0



Note: Decision rules in the developing country case. The left-hand plot gives the savings decision, while the right-hand plot presents the portfolio decision.

Figure 1.6 shows the dynamics of capital and equilibrium in the developed economy case with A_1. The upper part of Figure 1.6 gives the probability of good draws in equilibrium, n^*. It increases in k and approaches 1 from below. As shown in the decision rules, savings increase in capital and the portfolio shifts toward risky securities, if k becomes higher. The total amount of savings invested in risky securities increases in k, and based on the equilibrium condition, there is an expansion in the equilibrium measure of available securities; hence, the higher probability of good draws. If capital k is sufficiently high in this economy, full diversification is achieved, that is $n^* = 1$.

Figure 1.5: Decision Rules with A_1



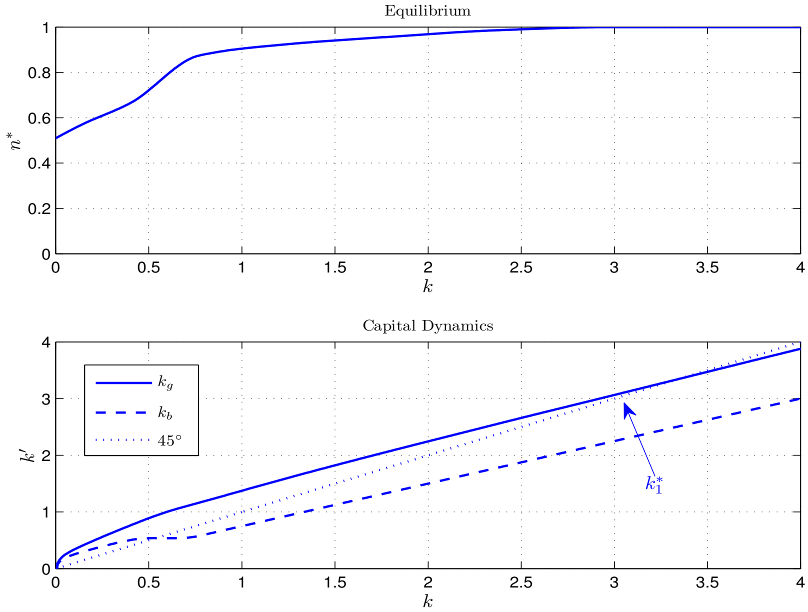
Note: Decision rules in the developed country case. The left-hand plot gives the savings decision, while the right-hand plot presents the portfolio decision.

The lower plot of Figure 1.6 gives two possible realizations of capital, k_g and k_b. The plot for k_g is increasing in capital, which describes the case of good draws. It does not only consist of undepreciated capital but also of returns to risky securities and the safe asset. k_b increases in k, when k is very low and decreases until a threshold where $\alpha^* = 0$ and then increases, following the line of $(1 - \delta) \cdot k$. As analyzed before, with the increase in k, the diversification opportunities improve and investment in the safe asset decreases until zero. Below that threshold, k_b comprises both undepreciated capital and return to the safe asset. Above that threshold, k is sufficiently high, and k_b only consists of undepreciated capital. In this situation, if bad shocks emerge in the intermediate sector, only undepreciated capital works as a buffer and becomes capital in the next period.[22]

The intercept of curve k_g with the 45 degree line gives the steady-state of the economy, k_1^*. In this steady state, full diversification is obtained and the probability of good draws is one. It means that if the economy reaches k_1^*, its capital is k_1^* in

[22]That is a scenario which does not occur in Acemoglu and Zilibotti (1997). In their model, full depreciation implies that agents invest all their savings in risky assets, only if the financial market is fully developed. Therefore, another implication of their model is that a reasonably rich economy could revert back to a really poor economy right before it becomes fully developed, with a small probability. Introducing undepreciated capital helps us provide the economy with an additional buffer and avoid the unrealistic implication.

Figure 1.6: Developed Economy with A_1



Note: Developed country case without exogenous shocks. The upper plot gives the equilibrium probability of a good draw. The lower plot gives two possible realizations at the end of the period and a 45 degree line. The dashed line stands for k_b, and the solid line for k_g.

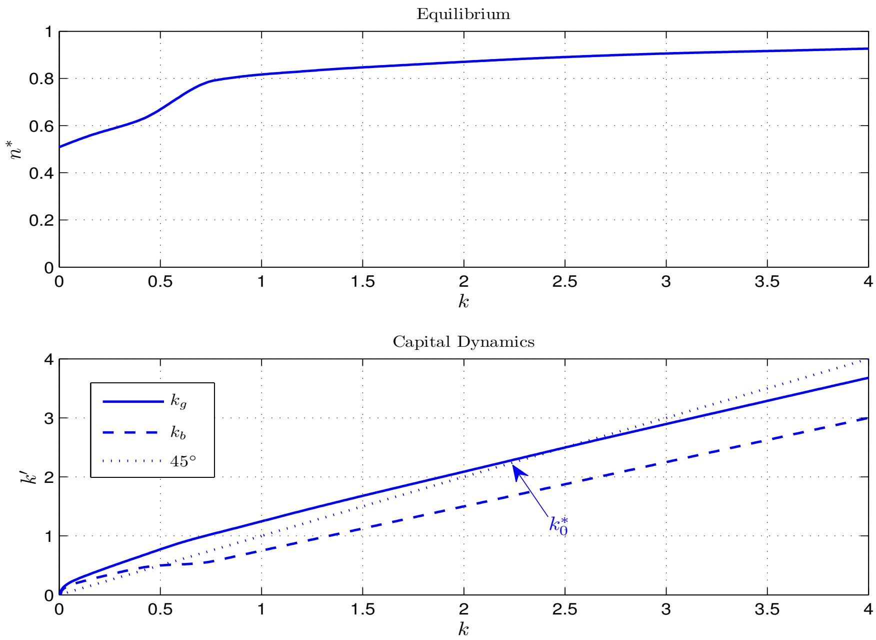
the next period with probability one. That is the situation which mimics standard neoclassical growth models.

Similarly, in the developing economy case with A_0, the probability of good draws in the intermediate sector increases in k and both k_g and k_b in Figure 1.7 present a similar shape as in Figure 1.6.

The intercept of curve k_g with the 45 degree line in Figure 1.7 gives the "pseudo steady-state" of the developing economy, k_0^*. In this case, the pseudo steady-state level of capital is lower than its counterpart in the developed economy with higher productivity, A_1. More importantly, full diversification is not achieved at this pseudo steady state. The probability of good draws is lower than one. With a positive probability, $k^{'}$ shifts onto the curve of k_b.

A comparison between Figure 1.7 with lower A_0 and Figure 1.6 with higher A_1 shows that the increase from A_0 to A_1 shifts the curves of $n^*(k; A)$ and $k^{'}(k; A)$ upwards. The following subsection shows that the dynamics of the two economies differ substantially.

Figure 1.7: Developing Economy with A_0



Note: Developing country case without exogenous shocks. The upper plot gives the equilibrium probability of a good draw. The lower plot gives two possible realizations of capital at the end of the period and a 45 degree line. The dashed line stands for k_b, and the solid line for k_g.

Consumption Dynamics and Amplification Effect

The developing economy analyzed in the last subsection most of the time exists in a world with a less fully diversified financial market. Most of the time, it also holds that $k \leq k_0^*$,[23] because there is always a positive probability that the economy is hit by bad shocks. Even without exogenous TFP shocks, the capital stock switches between the curves of k_g and k_b. Therefore, the output also jumps up and down substantially as does consumption.

The previous subsection also shows that without any TFP shocks, the developed economy would achieve full diversification and remain in the steady-state, where both consumption and output are constant.

It is interesting to compare the developed economy case with standard stochastic growth models. It is not difficult to imagine that with exogenous shocks, the developed economy fluctuates around the deterministic steady state and behaves similarly to a standard stochastic growth model. The subtle and non-trivial difference is that a sequence of bad TFP realizations might possibly decrease capital to such a low level that full diversification cannot be afforded and the economy

[23]If the economy starts with a capital $k_0 < k_0^*$, its capital will never exceed k_0^*.

could be hit by bad draws. Thus, on one hand, the model predicts frequent "small recessions" , which are driven by the exogenous TFP shocks around steady-state; on the other hand, it also predicts that deep and persistent recessions could happen in developed countries. Those rare events are driven by the combination of a sequence of low realizations of TFP shocks and the bad draws emerging from the financial market.

The endogenous diversification channel does not only provide an important source of uncertainty, which makes output more volatile, it also amplify the uncertainty in a certain period through capital accumulation. Thus, it has interesting implications for consumption behavior as well. The mechanism of *amplification* is that the probability of good draws is an increasing function of k. It means that, for a given level of capital k, if the economy happens to be hit by a good draw, the capital will be higher in the next period and the diversification opportunities will expand. Accordingly, there will be a higher probability of once more being hit by a good draw in the period after the next. This results in higher output in the next period and possibly an even higher output in the following periods. In contrast, if the economy happens to be hit by a bad draw, the capital will be substantially reduced in the next period and the degree of diversification will also shrink. The probability of being hit by a good draw decreases further. It results in a lower output in the next period and possibly even lower outputs in future periods.

This amplification channel is absent in the developed economy most of the time, since the market is fully diversified and $n(k; A_1) = 1$. In the case of the developing economy, in contrast, this channel plays an important role in consumption dynamics. As analyzed above, output becomes more volatile in this case, because of the shocks from incomplete financial market. Consumption will be even more volatile in the developing economy, in response to both current income of higher volatility and to the endogenous diversification channel, which affects future consumption opportunities. In other words, since capital k contains additional information about future output, consumption would respond to the amplification effect, while output would just keep track of changes in capital levels.

Moreover, TFP shocks, although assumed to be exogenous, would interact with endogenous shocks from the financial market. The mechanism of *interaction* works as follows. A good realization of the TFP shock enhances the productivity of the economy. It results in higher savings, which implies more aggregate investment in risky securities. It helps the economy improve the diversification opportunities, and therefore increases the probability of good draws. Therefore, even though the TFP shock itself is transitory, it could be amplified by interacting with the endogenous diversification channel and have a persistent effect on future output. In this sense, the TFP shocks would endogenously have a higher persistence.

The model predicts that the output gains during expansion can be larger in the developing economy: A sequence of good TFP shocks could interact with the diversification channel and lead to substantial output gains in the developing economy. In other words, booms are reinforced and become stronger through the interaction channel. This prediction is well consistent with the empirical findings in Calderón and Fuentes. (2006), which find that output gains are larger in lower income countries, during the trough-to-peak phase.

Since the exogenous TFP shocks are endogenously more persistent through the interaction channel, consumption in the developing economy is expected to respond to this and becomes more volatile. Once more, this channel plays a minor role in the developed counterpart since the financial market is complete most of the time. It implies that the ratio concerned should be relatively higher in developing countries.

1.3 Numerical Analysis

In this section, I will outline the solution algorithm and the simulation procedure (see the Appendix for a detailed algorithm). Similarly to most stochastic dynamic general equilibrium models, the system of equations (equations (1.1) and (1.2)) does not have any analytical solution, except the special case. I solve the model numerically by exploring the recursive formulation.

The numerical exercise posts interesting challenges. First, the function which maps aggregate states to the equilibrium measure of available securities needs to be solved endogenously in equilibrium.[24] Second, the endogenous shocks could substantially shift the capital stock in the economy within a broad ergodic set. Methods with local approximations cannot be used. Decision rules must be solved for over a large range in which the curvature of the decision rules changes dramatically. Third, another difficulty is due to the kink-shape of the portfolio decision and thus, I have to choose the approximation methods wisely. Finally, the inequality in one of the Euler equations brings in an additional difficulty.

Two steps are taken to solve the general equilibrium problem. First, I take an educated guess for $n(k, A)$ and solve the two functional equations with a root-finding (Broyden) method. The solved decision rules, together with the guess, would imply whether the financial market equilibrium condition holds. Second, if the equilibrium condition does not hold, update the function of $n(k, A)$ until the equilibrium condition is met. I simulate the economy for 11000 periods to compute the average statistics.

1.3.1 Calibration

In this subsection, I outline the calibration strategy. Table 3.3 summarizes the values and describes the parameters.

[24] Unlike the Aiyagari-type model, where only one factor price needs to be updated to ensure the equilibrium, the whole function in this model must be updated until the equilibrium condition is met.

I calibrate the model economy with different average productivity parameters, that is μ_0 and μ_1, to match certain moments in the data for developing and developed countries. I normalize the average productivity to be 1 for developing countries, or $\mu_0 = 0$. μ_1, which characterizes the average productivity level in developed countries, will be calibrated. Except for that, all other parameters are common to both groups of countries.

The remaining parameters are those characterizing preference and technology and those related to the financial market.

Regarding preference and technology, I parameterize the model using standard data in the growth literature. I use the standard $CRRA$ utility function, where the risk aversion parameter is chosen to be 1.5. The discount rate β is standard from the literature, 0.96. The capital share is set to be 0.3, which is also common in growth models. I choose the annual depreciation rate to be 0.10. Values of $\rho = 0.95$ and $\sigma_z = 0.02$ are widely used in the literature. The AR(1) shock process is approximated by a Markov chain, using the Tauchen method (Tauchen 1986).

Regarding the parameters characterizing the financial market, the gross return to safe assets, r, is normalized to 1, and the gross return to risky projects, R, and the minimum requirement parameters, γ and D, will be calibrated.

I choose to match long run average saving rates (s/y) and output growth volatilities (σ_y) in both developing and developed countries by choosing these four need-to-be-calibrated parameters (see Table 1.3).[25] I compute the average saving rates for developing and developed countries and find there to be a statistically significant difference between these two groups. It is also interesting to let the model deliver the correct output growth volatilities and observe if consumption growth volatilities are sufficiently close to the data.

The parameters are jointly mapped to the targets. An increase in D or a decrease in γ would shift the curve of minimum requirement upwards and thus, the economy is more likely to be constrained by micro-level project indivisibility. The volatility of output growth in both economies will be higher. A higher R implies that the return to the risky portfolio is higher, which would induce a higher saving rate in both economies. A larger value of μ_1 would only make the developed economy less constrained and reduce its volatility level. A higher μ_1 also implies that the risky portfolio is "safer" and its savings rate is therefore higher, since the financial market is more complete, everything else equal.

1.4 Results

This section discusses the basic results from the simulations. I present the representative simulation series for the experiments of developing and developed countries

[25] The saving rate refers to the "gross domestic saving rate " in The World Development Indicators data. I compute the average saving rate for each country from 1960 to 2007 and compute the group mean for each group.

Table 1.2: Baseline Parameters

Parameter	Economic interpretation	Value
σ	CRRA risk aversion	1.5
β	Annual discount rate	0.96
η	Capital share	0.30
δ	Depreciation rate	0.10
ρ	Shock persistence	0.95
σ_z	Shock standard deviation	0.02
r	Return to safe asset	1.00
R	Return to risky securities	1.055
D	Largest minimum size	1.70
γ	Minimum size parameter	0.16
μ	Log of average of productivity	0.605

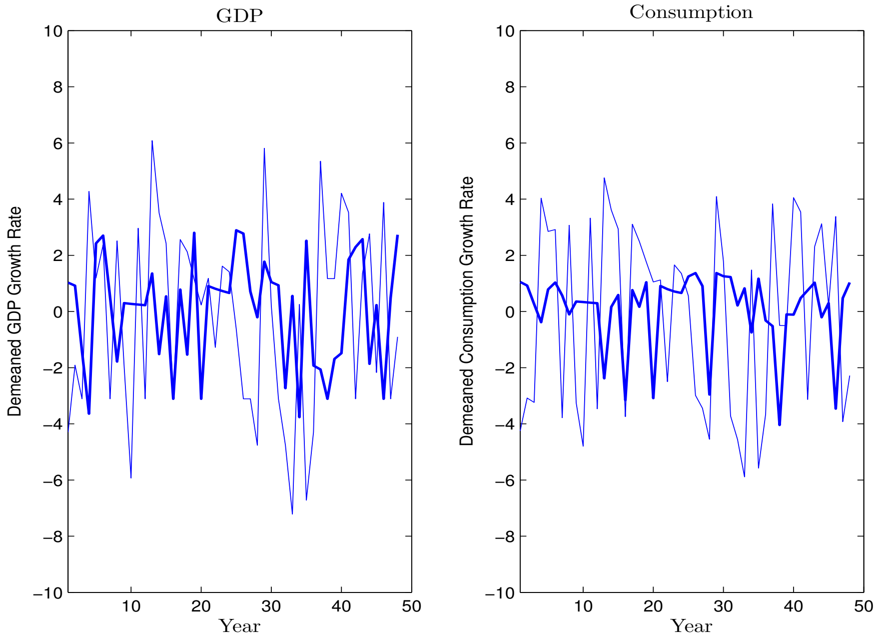
Source: Standard and calibrated parameters

Table 1.3: Moments to Match

	Saving Rate	Model
Developed countries	$s/y = 0.25$	0.29
Developing countries	$s/y = 0.19$	0.176
	Growth Volatility	Model
Developed countries	$\sigma_y = 2.403$	2.43
Developing countries	$\sigma_y = 4.503$	4.48

Source: WDI (1960 - 2007) and simulations in Figure 1.8.[26] In both experiments, the exogenous shock processes are set to be exactly the same.

Figure 1.8: Representative Simulation



Source: Simulation data. Demeaned growth rates of output and consumption are presented on the left-hand side and the right-hand side, respectively. The thicker curves are from the experiment of the developed country case while the regular curves are demeaned growth rates from the experiment of the developing countries. In both experiments, the persistence and variance of the exogenous shock processes are the same.

The excess consumption growth volatility pattern is quite clear. The left-hand side graph presents the demeaned output growth rates of these two experiments. The thicker line represents the developed economy experiment, while the regular line stands for the developing economy experiment. It is observed that the developing economy experiences a more volatile output growth. It is even more obvious that the volatility of the consumption growth rate is higher in the developing economy than in the developed economy. As compared to output growth rate volatility, consumption growth rates seem to be more smooth in the developed case. In the experiment for the developing economy, the consumption growth rate is much less smooth.

[26] A series of realizations of 47 periods is randomly picked from each simulation.

Table 1.4: Excess Volatility

Type	σ_c	σ_y	σ_c/σ_y	$(\sigma_c/\sigma_y)^{data}$
Developed countries	1.87	2.43	0.768	0.896
Developing countries	4.53	4.48	1.01	1.197

Source: Simulation data and WDI data(1960-2007).

Table 1.5: Development Level

	μ_0	μ_a	μ_b	μ_c	μ_1
μ	0.00	0.25	0.45	0.55	0.605
σ_c/σ_y	1.01	0.96	0.93	0.84	0.768

Source: Simulation data.

1.4.1 Basic Results

The main results are summarized in Table 2.5. I compute the average standard deviation of consumption growth rates from the simulation data. The first column reports the results from the two experiments. I also compute the ratio of consumption growth volatility to output growth volatility in the third column.

First, the first and second columns show that the negative relationship between development and growth volatility exists in the model and it is even more pronounced in the case of consumption growth volatility.

Second, the third column reveals that, in this model, the ratio of consumption growth volatility to output growth volatility is substantially higher in the developing economy. It is approximately 1 in the experiment for the developing economy, while the ratio in the developed economy case is below 0.77. The pattern found in the model is well consistent with the empirical findings in the data and in Kose, Prasad, and Terrones (2003).

This result confirms the intuition that the higher is the development level, characterized by a higher μ in the model, the less likely would the micro level constraint be to bind. The financial market will be more complete, shocks from the intermediate sector would play a minor role, and the ratio in question would be lower. In this model, it is observed that the lower productivity level is associated with both higher output and consumption growth volatilities, keeping the variance and persistence of exogenous shocks unchanged. More interestingly, consumption growth volatility is disproportionately higher, relative to output growth volatility. These results are not expected in the standard stochastic growth models.

The model also predicts that if an economy switches from a lower level of development to a more advanced level, both consumption and output growth volatility should be lower and consumption growth volatility should drop more quickly. Table 1.5 gives results from a series of experiments, which use intermediate values of μ, that is μ_a, μ_b and μ_c, between μ_0, the normalized value for the developing group, and μ_1, the calibrated value for the developed group. It is shown that the ratio of σ_c/σ_y decreases steadily with the increase in the value of μ.

Finally, although this model successfully generates the empirical pattern, there are still notable differences between the model and the data. Volatilities of consumption growth of both types of economy are lower in the model than their counterparts in the data. The model under-predicts the consumption growth volatility in both types of economy, since there are only two sources of uncertainty in the model.[27]

1.4.2 Measuring TFP Properties

With the standard growth accounting method, information about the properties of TFP shocks can be extracted from the Solow residual, by assuming one unit of savings in each period to be transformed into one unit of gross investment. The model assumes there to be an intermediate sector where the transformation process is stochastic, due to the fact that the financial market is not necessarily fully diversified. If I make use of information about capital which is actually used in the final goods production, the Solow residual could correctly help identify the TFP shock process. Therefore, the properties of the TFP shock should be identical in simulations for developing and developed economies, since I impose the same exogenous shock process on both experiments. However, if constructing the capital sequence by ignoring the incomplete financial market structure, the results will be biased.

Suppose that the model proposed in the paper exactly captures the reality, but that one tries to understand the data produced by the model, with a "misspecified" model, where an exogenous difference between the TFP shock processes for the two types of economy is assumed. If the estimation concludes there to be a difference between the TFP processes of the two groups, the observed difference should be attributed to the ignored endogenous diversification channel.

One of the important findings in Aguiar and Gopinath (2007) is that permanent shocks are relatively unimportant in industrial economies, as compared to lower income emerging economies.[28] This section evaluates if this model with an endogenous diversification channel has the ability to generate the observed difference. Toward this end, I use the same approach, proposed by Aguiar and Gopinath (2007) to analyze the Solow residual from simulation data.

[27]However, it is not hard to imagine that other channels of uncertainty would increase the volatility level of consumption growth. Empirical evidence (e.g. Easterly, Islam, and Stiglitz 2000) also shows that fiscal policy, public consumption and nominal shocks all help to increase the volatility in both developing and developed countries. The international sector can contribute another source of uncertainty, which exposes the economy to external shocks (e.g. Neumeyer and Perri 2005).

[28]In their model, they assume the permanent shock to be an accumulative product of "growth shocks" and growth shocks to be positively correlated. They find that the Solow residual in lower income countries is more volatile and the permanent component

I conduct this exercise in the following way. Suppose that one tries to analyze the simulation data, with an otherwise standard stochastic growth model, augmented by exogenous permanent shocks. The production function is assumed to be Cobb-Douglas, producing final goods with capital and one unit of labor.

$$Y_t = e^{z_t} K_t^{\eta} \left(\Gamma_t L_t\right)^{1-\eta}$$

There are two types of shocks. The first is z_t, which is a standard $AR(1)$ process.

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z$$

The second, Γ_t, is used to represent the cumulative product of white noise. Specifically,

$$\Gamma_t = e^{\varepsilon_t^g} \Gamma_{t-1} = \prod_{s=0}^{t} e^{\varepsilon_s^g}$$

where ε^g is an innovation from a normal distribution with zero mean and standard deviation σ_g.

To directly estimate the underlying shock process, one wants to make use of information from Solow residuals. Savings in simulation data are assumed to be converted into investment one for one in a close economy. Given the initial capital k_0 from the simulation, a sequence of pseudo capital can be constructed. With information on output sequence, the sequence of the pseudo Solow residual can be backed out from the final goods production function.

To extract information on permanent shocks from the pseudo Solow residual, one makes use of the standard method proposed by Beveridge and Nelson (1981), which implies that if following an $I(1)$ process, the log of the Solow residual can be decomposed into a random walk component and a transitory component.

$$sr_t = \tau_t + z_t$$

where sr_t is the log of the Solow residual. Moreover, τ_t represents random walk and follows the below process,

$$\tau_t = \tau_{t-1} + (1-\eta)\, \varepsilon_t^g$$

Following the method proposed by Cochrane (1988), the variance of the permanent component is estimated with the below approximation,

is larger than it is in developed countries. They further argue that, between the two groups, the ratio of the standard deviation of permanent shocks to the standard deviation of mean reverting shocks is different. In the misspecified model, the permanent shock to be identified is an accumulative product of white noise, which can be interpreted as uncorrelated growth shocks.

$$\lim_{M\to\infty} M^{-1}Var\left(sr_t - sr_{t-M}\right) = \sigma^2_{\Delta\tau}$$

where sr_{t-M} is the M period lag of log of Solow residual. The estimation is sufficiently accurate, conditional on M being sufficiently large. In practice, it is difficult to choose M to be sufficiently large due to the limitation of the data length. However, this exercise is not constrained by data length, since I can simulate the model for a sufficiently long period of time. I choose M to be unusually large, higher than what would be chosen in practice. I choose M to be 1000, 2000, 3000, respectively, in different estimations.

The critical point of this exercise is to discover the difference of permanent shocks between experiments of developing and developed economies. Towards this end, I look at the ratio of the standard deviation of permanent shocks in the developing economy to that in the developed economy. The essential finding is that the ratio is quite stable and roughly 1.8, despite the choice of M.

$$\frac{\sigma^{developing}_{\Delta\tau}}{\sigma^{developed}_{\Delta\tau}} \approx 1.8$$

This finding shows that if using the standard growth accounting method to construct the Solow residuals, it would be concluded that there is a difference in the measured TFP processes. However, the data are generated by the true model, where no difference in the TFP process is exogenously assumed. In other words, the diversification channel in this model could at least partially explain the observed difference in the measured TFP processes of developing and developed countries.

1.5 Conclusion

Mounting evidence shows that consumption growth exhibits an excess volatility in developing countries. This pattern has been documented in several recent research papers.

This research shows that the lack of diversification opportunity in developing countries could account for the empirical regularity. This paper models this connection based on Acemoglu and Zilibotti(1997)'s classical contribution in this field. Numerical results show that the model is well in line with the empirical regularity documented in different datasets, although the volatility of consumption growth in both groups is obviously lower in the model than in the data. Interestingly, in this model, even though both types of economy are subject to exactly the same exogenous shock process, the only difference in average TFP translates into the excess consumption growth volatility pattern.

In this research, it is argued that the endogenous difference in the financial development of both types of economy could help explain observed differences in TFP processes. When the financial development is low, stochastic shocks to investments and TFP are endogenously more volatile and more persistent. This model also has an implication for estimating TFP processes in developing countries: The estimation might be biased if we ignore the endogenous diversification channel.

Chapter 2

Mortgage Loans, the Decline of the Household Saving Rate and the Increase in Risk-Sharing

Chapter Summary

Between 1980 and the beginning of the current century, the U.S. economy experienced: (i) a sharp decline in the personal saving rate, which was associated with a consumption boom, (ii) an increase in mortgage debt, (iii) a rise in the homeownership rate and (iv) an improvement in risk-sharing. In this paper, we analyze to what extent the deregulation of housing finance - i.e., the increased availability of refinancing opportunities and the decrease of effective downpayment requirements -accounts for these trends. To study the impact of the financial deregulation, we implement housing and mortgage loans in an otherwise standard quantitative life-cycle model. The model is consistent with the increase in both net mortgage debt and the homeownership rate observed in the data; it delivers one third of the increase in the ratio of consumption to personal income. Refinancing provides an additional risk-sharing channel by making housing wealth more liquid. As a consequence, households reduce their precautionary saving, take out more collateralized loans and increase both housing and nonhousing consumption. We find that lower downpayment requirements only amplify the effects of refinancing but they do not cause the observed changes in the aggregate trends.

Key words: Mortgage Loans, Financial Deregulation, Risk-Sharing, Household Consumption and Saving, Incomplete Markets

JEL classification: E2, D5, D9

2.1 Introduction

Between 1980 and the beginning of the twenty-first century, the U.S. economy experienced an 7 percentage point decline in the personal saving rate, and an increase in the consumption share of personal income by 5.5 percentage points. Moreover, the homeownership rate rose by 6 percentage points, while household collateralized debt increased substantially as well.

At the same time, mortgage financing in the U.S. has undergone substantial changes. As we will document later in this study, until the beginning of the 1980s, fixed-term, level-payment mortgage loans were basically the only way to finance a home purchase. One of the most important features of these types of contracts is that homeowners had to increase their share of home equity over the contract period. During an era of financial deregulation, financial instruments, which helped homeowners to access to home equity became relatively inexpensive and widespread.

Our paper tries to quantitatively evaluate the impact of the change in housing financing on aggregate consumption, saving rate, debt holdings as well as homeownership rate. Given that home equity is the single largest position in most household's portfolios, and given that the vast majority of homeowners hold mortgage debt, it is plausible that the changes in the mortgage market are responsible for the documented trends.

We find that in total, mortgage market deregulation accounts for roughly one third of the increase in personal consumption share and the substantial increase in mortgage debt. Our model is also consistent with the increase in the homeownership rate that was observed in the U.S. economy. Moreover, we find that refinancing opportunities play a key role in financial liberalization. The low downpayment requirements amplify the effects of refinancing but turn out to be relatively less important.

Our findings are important for the following reasons. Previous literature has not fully examined the extent to which deregulating mortgage markets accounts for the aggregate trends in consumption and saving. Parker (1999) argues that even if financial deregulation is fully responsible for the rise of household debt, its impact would be too small to account for the increase in aggregate household expenditures. On the other hand, Feldstein (2008) argues that the high level of of mortgage refinancing with equity withdrawal is the primary reason for the low U.S. saving rate and the current account deficit. Recent work by Mendoza et al. (2007) provides further support for Feldstein's hypothesis.[1] Against the background of this discussion, our work attempts to evaluate the different hypotheses. Since there is little doubt that the documented substantial trends are at stake in the U.S. economy, our results have important implications for optimal policy design.

[1] Mendoza et al. (2007) show that that differences in saving across countries that result from differences in financial market development can lead to global imbalances in the magnitude that is currently observed.

We construct a life-cycle model to shed further light on the role of mortgage market deregulation that accounts for aggregate trends in the U.S. economy. In our model, households receive utility from consuming housing services and nondurable goods. Homeownership is endogenous, and housing services can be acquired through owner-occupied housing or on the rental market. Moreover, households are subject to idiosyncratic, labor income earnings risk. The financial market is assumed to be incomplete; households can only self-insure against earnings shocks by saving in a noncontingent bond.

We model mortgage market deregulation by comparing two different mortgage regimes that we take as a stylized representation of the mortgage loan contracts observed in the U.S. economy. In the first regime, homeowners are forced to repay their obligations, irrespective of the income shocks. Thus, we say that this type of contract gives rise to "forced saving"in home equity. Moreover, equity accumulated in the home cannot be accessed for consumption smoothing purposes later. We also define this regime as a "traditional mortgage". The modeling of mortgages resembles the traditional, fixed-term level payment mortgage loan. See also Chambers et al.(2009a, 2009b) and more recently Nakajima and Telyukova (2009) for similar approaches to modeling mortgage loans. In the second regime, homeowners can avoid pre-fixed repayment by periodically refinancing. In other words, homeowners can revise their mortgage payment contract by negotiating payment schedules and/or altering the amount of repayment.[2]

Notice that the key difference between the first and second regime is that in the latter, households are not forced to accumulate equity in their home beyond the initial home equity at purchase (the downpayment requirement).

Moreover, an additional consequence of mortgage market deregulation was the decline in the downpayment requirements. We characterize this aspect of deregulation by using a lower downpayment ratio in the second regime.

In our experiments, we take an economy where only traditional mortgages are available as being representative of the pre-deregulation period (beginning of the 1980s). The situation in the post-deregulation period is approximated by the second mortgage regime with a lower downpayment requirement.

What we label as the second regime has long been the standard choice of modeling housing finance in the literature (See, for example, Díaz and Luengo-Prado 2009, Iacoviello and Pavan 2009, Fernandez-Villaverde and Krueger 2010, Yang 2009, Li, Liu, and Yao 2009, Kiyotaki, Michaelides, and Nikolov 2007, Hintermaier and Koeniger 2009 and Scoccianti 2009, among others.).

Compared to the previous literature, the advantages of our models are twofold. Firstly, our set-up allows us to show the *net effects* of the refinancing options. Secondly, it helps us decompose the total impact of financial deregulation into changes that result from lower downpayment requirement and changes that occur

[2]Both of these ways of refinancing help homeowners to change the payment flow.

because mortgage loans became more flexible.[3] The previous literature has shown that lower downpayment requirements lead to more collateralized debt and higher homeownership rate. In addition, we explicitly examined the effect of refinancing opportunities alone on both mortgage debt and homeownership rate.

Our results show that by deregulating mortgage markets (i.e., by moving from regime 1 to regime 2), consumption, debt and homeownership rate increase and as a result savings fall. The driving force is the additional risk-sharing channel provided by the availability of refinancing opportunities.

Since markets are incomplete, the only way for households to *self-insure* against adverse income shocks is by accumulating a noncontingent bond (precautionary saving). This is the case in regime 1 as well as in regime 2. However, in regime 1, households are forced to increase their home equity as the mortgage matures. We show that forced-home-equity-saving actually increases the demand for precautionary saving. Intuitively, forced-home-equity-saving imply additional committed expenditure for all possible states of nature. This mechanism gives homeowners additional incentive to hold more precautionary savings in order to smooth out nonhousing consumption.

However, when mortgage deregulation takes place and mortgage market structure moves from regime 1 to regime 2, homeowners are not forced to build up home equity over time by following the contracted repayment plan. Moreover, homeowners can access home equity they accumulated in the past easily and use home equity to insure themselves against negative income shocks. The demand for precautionary saving decreases. Therefore, resources will be released for nonhousing consumption.

This implies that households can choose to raise more debt. Homeowners can borrow against home equity, when negative income shocks occur. Moreover, anticipating that home equity can be accessed, households are more willing to accumulate housing stock, compared with households in the traditional mortgage regime. That increases the debt holding even further, since more households can borrow and purchase homes. As a result, both the homeownership rate and housing consumption have increased.

All these effects are amplified when the downpayment requirements become less strict as well. It should be noted, however, that our results indicate that removing the forced-home-equity-saving component from the mortgage loan is more relevant for analyzing the effects of financial deregulation than lowering the downpayment requirement.

It is important to notice that we keep house prices constant in our analysis. Thus, the increase in consumption in this model economy does not stem from

[3] Starting with the seminal work of Jappelli and Pagano 1994, the previous literature has mainly used the downpayment requirement as a measure for the restrictiveness of mortgage contracts. Consequently, financial liberalization is modeled as an event that leads to lower downpayment requirements (See, for example Iacoviello 2008 or most recently Favilukis et al. 2010 and Scoccianti 2009).

the "wealth effect." [4] The evidence on the wealth effect is mixed in empirical literature. Carroll et al. (2006) estimate that an one-dollar increase in housing wealth raises consumption by about nine cents, while Lettau and Ludvigson (2004) conclude that the marginal propensity to consume out of housing wealth is close to zero.

In this paper, we do not model fluctuation of housing prices and focus on the impact of refinancing opportunities to explain the consumption boom. It is a more important aspect, since homeowners must have access to home equity before they can take advantage of the increased housing value. Indeed, Doms et al. (2008) provide empirical evidence for the fact that the linkage between housing wealth and nonhousing consumption increased after mortgage market deregulation, suggesting that refinancing opportunities play an important role not only for housing consumption, but also for nonhousing consumption.

Hurst and Stafford (2004) provide further empirical evidence in support of our hypothesis by showing that households indeed refinance their mortgage for consumption-smoothing purposes.[5] Their empirical evidence lends direct support for one of the main mechanisms highlighted in our model: mortgage refinancing plays an important role as a consumption smoothing mechanism, besides the use of standard precautionary savings.

Precisely and intuitively, refinancing provides an additional channel of risk-sharing. It allows for state-contingent saving, in the sense that households only save when income shocks are positive. In the presence of adverse income shocks, households could borrow by using their home equity as collateral. In contrast, the traditional mortgage payment plan forces homeowners to save in home equity in all states of nature.

It should be noted that in the refinancing regime, households rely less on the precautionary savings for insurance. Or in other words, they substitute the home-equity risk-sharing channel for the standard precautionary-saving, self-insurance channel. It is not clear if the over all risk-sharing opportunity would improve or not. However, our results indicate that total risk-sharing opportunities for homeowners improve in the refinancing regime.

Similarly, Favilukis et al. (2010) also concluded that housing finance deregulation increases the risk-sharing opportunities in a model with housing production. In their model, they also showed that both lower downpayment ratios and lower transaction costs increase households' access to credit, which helps households to insure better their idiosyncratic risks. However, they focus on the period between 2000 and 2006, where they implicitly assume that refinancing was available. In contrast, we compare the period between 1980 and the beginning of new century,

[4] According to the permanent-income hypothesis, consumption adjusts to the unexpected changes in housing wealth induced by changes in housing prices.

[5] Previously, the housing literature mainly focused on the "financial motivation" of mortgage refinancing. And house price and mortgage rate changes were considered to be the main driver for refinancing activities.

where mortgage refinancing became increasingly popular. Although we also conclude that lowering the effective downpayment ratio helps to increase risk-sharing, we highlight that the change in mortgage structure alone played a major role in increasing risk-sharing opportunities.

Hryshko et al. (2009) argue that house prices appreciation helped homeowners to extend their borrowing capacities because the value of their collateral has increased. We assume that housing prices are constant and stress that the availability of refinancing itself can provide sizable increase in risk-sharing opportunity, even when housing prices do not change. It should be noted that increasing house prices can only provide better risk-sharing in combination with better opportunities to access home equity. Otherwise, homeowners do not benefit from changes in their housing wealth.

Thus, our paper also contributes to the literature that argues that risk-sharing opportunities have improved over the last decades. Krueger and Perri (2006), for example, argued that the decoupling between income and consumption inequality points to the fact that market completeness, i.e. the ability of private financial markets to insure risks, has improved over time. We propose a specific channel through which financial markets could have provided better risk-sharing opportunities and quantify its relevance. In another important contribution, Heathcote et al. (2008) depart from the bachelor model of household formation and argue that risk-sharing within the couple can explain the decoupling between earnings and consumption inequality. We view their explanation as complementary to ours.

The rest of the paper is comprised of the following sections: In the section 2.2, we document the substantial changes in the U.S. housing market over the last decades, as well as the institutional background for the changes. We also present evidence of the decline in saving rates, the consumption boom and the increase in debt during that period. Section 2.3 presents a simple life-cycle model with analytical solutions. It shows the main mechanisms, by which the refinancing opportunities affect the saving rate and risk-sharing among households. Section 2.4 outlines quantitative models that compare the two regimes. Section 2.5 details the calibration strategy and discusses the main results. Section 2.6 concludes with recommendations for possible future research.

2.2 Institutional Background: Deregulation, Mortgage Debt, and Saving Rate from 1980-2000

In this section, we first outline the institutional origins of what is commonly called "traditional mortgages", i.e. long-term amortized mortgages that require the mortgagee to take an initial equity share at the time of purchase and to accumulate further equity as the debt amortizes (typically over a period of 15 to 30 years). We then present the emergence and popularity of refinancing opportunities in the last decades.[6] We also document the substantial increase in the household debt holding and the decline in the household saving rate, as well as the associated "consumption boom", during the period from 1980 to the beginning of the current century in the U.S. economy.

2.2.1 Origins of the Long-Term Amortized Mortgage Structure

The vast majority of homeowners (about 90%) acquire their homes by using long-term amortized mortgages mortgages, which is considered the "standard" loan product.

Amortized mortgages originated with the New Deal regulations, with particular in the Federal Home Loan Bank Act of 1932 and the Home Owners' Loan Act of 1933. These regulations reflected the desire of the Roosevelt administration to reduce the likelihood of a mass default, which occurred at the beginning of the Great Depression (Campbell and Hercowitz 2006 and 2009). In particular, the "forced saving"component of long-term amortized mortgages (i.e., the fact that homeowners were forced to raise their equity in their homes as the loan amortizes) was seen as a way to reduce the possibility of systemic default. Before, interest-only, periodically refinanced mortgages were common, which allowed homeowners to hold a very low equity share in their homes.

Depression-era regulation was also meant to insulate the mortgage market from fluctuations in other financial markets (Campbell and Hercowitz 2006 and 2009). Prior to the 1980s, mortgage loans were almost exclusively issued by thrift institutions (savings and loans). Regulation constrained savings and loans to raise most of their funds needed to issue mortgage loans by using short-term deposits. This is referred to as the "maturity mismatch"problem, as lenders were forced to finance long-term mortgages with short-term liabilities. Moreover, thrift institutions faced usury laws and interest-rate caps ("Regulation Q") which restricted the conditions to which they could borrow and lend (Campbell and Hercowitz 2006 and 2009 and Gerardi et al. 2006).

2.2.2 Structural Change in the Mortgage Market and the Emergence of Refinancing Opportunities

The high inflation at the end of the 1970's and the beginning of the 1980's made the New Deal financial regulations untenable. The main reason for the failure was

[6] We draws heavily on Campbell and Hercowitz(2006,2009) as well as on Gerardi et al. (2006).

the maturity mismatch on the balance sheet of the savings and loans institutions. The fact that savings and loans financed mortgage loans with short-term deposits made mortgage lending unprofitable, because nominal interest rates on mortgages were fixed and relatively low compared to the high, nominal interest rates that were required to attract deposits (Gerardi et al. 2006).

Congress and Presidents Carter and Reagan responded with the Monetary Control Act of 1980 and the Garn-St. Germain Act of 1982. As a result, usury ceilings, interstate banking prohibitions, limits on branching, and Regulation Q, which capped deposit rates and forbade banks from paying interest on checking deposits, were abolished. Moreover, state laws that constrained the types of mortgage products originators could offer were preempted (Gerardi et al. 2006).

As a result, a process which is often referred to as "financial innovation" began. This has changed the housing finance system dramatically, increasing the menu of mortgages available to homeowners considerably. Lenders now offered borrowers much more flexible repayment schedules.

Instruments for avoiding forced saving already existed before financial deregulation. One could cash-out previously accumulated home equity either by taking a second mortgage or a home-equity loan or by refinancing the debt with a loan exceeding the current mortgage balance. However, lack of competition made these products prohibitively expensive. In contrast, deregulation eliminated legal barriers to entry and was so considerably more competitive (Bennett et al. 2001). In addition to deregulation, advances in information processing technology also helped to lower costs by streamlining the mortgage application process (Bennett et al. 2001).

2.2.3 Mortgage Refinancing over the Last Decades

After the financial deregulation, refinancing activities have been increasingly popular, and homeowners have been using refinancing as a tool to smooth their consumption when negative income shocks occur.

Using various waves of the SCF (Survey of Consumer Finances), Campbell and Hercowitz (2009) document a sharp increase in refinancing activities after the deregulation (See Table 2.1.). In 1983, approximately 10 percent of mortgagees had refinanced. By 1989, this number had more than doubled, increasing again to 33 percent in 1992. In 1995, it reached almost 41 percent, and it was slightly higher than that in 1998 and 2001. Campbell and Hercovitz conclude that "mortgage refinancing went from atypical to commonplace in about 12 years "(Campbell and Hercowitz 2009, p.3).[7]

[7] Note that the reported numbers are likely to underestimate the true trend of refinancing activities. Campbell and Hercowitz (2009) measure refinancing activities by computing the share of those households for which the year of home purchase does not coincide with the year the oldest mortgage debt was issued. This identification strategy misses those households who pre-pay (i.e. repay their mortgage loan before maturity) as these households do not take out a new mortgage.

Table 2.1: Refinancing Activities and Effective Downpayment

Year	Percent of Mortgagees Refinanced	Average Equity/Value at Purchase
1983	9.9	22.6
1989	21.2	23.4
1992	33.0	20.9
1995	40.9	16.9
1998	42.3	16.4
2001	44.4	16.4

Source: From Survey of Consumer Finances
Campbell and Hercowitz (2009)

It is important to study the motives of households who refinance. Some may refinance their mortgage for consumption smoothing purposes, while others may simply want to take advantage of lower interest rates.[8] Interest rate movements and expected interest rates movements can explain some of the refinancing activity. However, as noted by Hurst and Stafford 2004, this explanation cannot account for fact that there is a significant share of mortgagees who refinance, even if interest rates are stable or even rising. Stanton (1995) notes that some fixed-rate mortgages are prepaid even when current market mortgage rates are above the household's contracted coupon rate, and some fixed-rate mortgages are not prepaid even when current market mortgage rates are well below the household's contracted coupon rate. Using the Panel Study of Income Dynamics (PSID), Hurst and Stafford (2004) find that many households refinance after a negative income shock. They conclude that the consumption smoothing motive is essential for understanding refinancing behavior.

Moreover, Campbell and Hercowitz (2009) also provide information about the development of effective downpayment ratios, i.e., ratio of the average equity relative to the value of newly purchased homes. Because of the financial deregulation, borrowers are allowed to take out second and third mortgages easier, and the effective downpayment ratio declines over time. This is indeed the case, as the second column of Table 2.1 shows.

In summary, the financial deregulation triggered innovations in the mortgage market, which helped an increasing number of homeowners to better access their home equity, both through loan refinancing and lower downpayment ratios.

[8] The latter motive, which we call "financial motivation "(Hurst and Stafford 2004), might be particularly important in an environment with volatile inflation rates that prevailed at the beginning of the 1980s. When current mortgage rates are below the existing mortgage contract rate, households have an incentive to replace their existing fixed-rate mortgage with one at a lower rate. The benefit to the household is a present value of wealth gain.

2.2.4 Household Mortgage Debt

Following the financial deregulation, mortgage debt began to grow. Mortgage debt relative to average labor income increased by more than 15 times, if we compare data from the 1983 and the 2004 Survey of Consumer Finances (SCF). In our calculation, mortgage debt includes primary mortgages, home equity loans, home equity lines of credit and labor income including wage income and income from self-employment.

It is interesting to see that mortgage debt grew not only in absolute terms, but also relative to the value of houses: the ratio of mortgage debt to the value of owner-occupied homes was 0.31 in 1982, 0.37 in 1990, and 0.42 in 1995. Since 1995, it has fluctuated around this higher level (see Campbell and Hercowitz 2009).

During the 1990s, not only mortgage debt, but also financial assets have grown rapidly. This suggests that many households may have used refinancing opportunities to leverage their financial market activities, instead of using these opportunities as a buffer for consumption. We thus also compute the *net debt*, defined as financial assets minus mortgage debt, if the difference is negative. If the households are in negative net financial position, they are net borrowers in this economy. We suspect that net borrowers are more likely to use mortgage debt for the purpose of consumption smoothing. Therefore, this measure is closer to the purpose of our study.

We find that net debt has grown as well, although less dramatic than mortgage debt. The fraction of the population that is in net debt has grown by 7 percentage points, and net financial debt relative to average labor income has grown from 123% to 160% between 1983 and 2004. This result can be interpreted as the average mortgage position in the economy.[9] Our definitions of debt, assets and income follow those used by Kennickell et al. in the SCF bulletin.[10]

These findings provide further evidence of the fact that mortgage debt rose and that it was increasingly used to finance consumption, either in form of housing services or in form of nonhousing consumption. As the next section shows, the increase in (net) debt indeed coincided with a rise in aggregate consumption.

2.2.5 The Decline of the Household Saving Rate and Consumption Boom

The U.S. economy experienced a decline in the household saving rate in the past decades. There are different definitions of household saving rate; however the decline is quite robust regardless of the definitions. For example, the *private*

[9] Notice, however, that our measure of income does not correspond to GDP.

[10] It should be noted that the 1983 is only partially comparable with the other SCF waves because of a change in sample design and in the questionnaire.

saving rate was at its post-1950 average level of 17.6 percent as recently as 1993, but it dropped to 13.1 percent at the very end of 1990's, reaching its lowest level in nearly 50 years.

At the meantime, it witnessed an even more dramatic decrease in the *personal saving rate*. Guidolin and Jeunesse (2007) document this change and argue that the decline in the personal saving rate is a very robust fact, despite measurement issues.[11] According to NIPA, it has dropped from 9.8 percent in the 1980 to roughly 2.9 in the 2000.

Although a wealth of evidence shows household saving rate declines substantially, National Income and Product Account (NIPA) definitions of saving rates do not directly correspond to the economics concepts, as Parker (1999) points out. Instead, it is simpler and more useful to exam the other side of the coin, namely the consumption share of national output. Consistent with those findings regarding the saving rate, it is documented that the consumption share of GDP has increased by 6 percentage points from 1980 to 2000. This increase itself is striking and seems even more pronounced by comparing the period from 1950 to 1980 where the consumption-GDP ratio was roughly constant. NIPA also shows that personal consumption expenditure accounted for 87.7% of personal disposable income in 1980 and the ratio has increased to 93.2% in 2000.

It is also important to note that NIPA data measures a broader definition of consumption, namely consumption expenditure. It includes both consumption (non-durable consumption plus services, including housing services) and durable goods expenditures (excluding housing). The expenditures on durable goods increased only by 0.1% from 1980 to 2000. Obviously, it is not sufficient to explain the consumption boom. On the other hand, data suggest that the share of non-durable consumption plus the services (including housing services) increases substantially. In other words, in order to account for the increase in substantial increase in the consumption share, one must explain why non-durable consumption and services increases substantially.

Even though a consensus of researchers believe that the dramatic decrease in the household saving rate is quite substantial, what has triggered this decline in the saving rate and increase in consumption share has remained a hot issue.

Among alternative explanations (wealth effect created by stock market bubbles, change in demographic structure, an increase in the discount rate, etc.), Feldstein (2008) hypothesizes that mortgage refinancing with equity withdrawal is one of the major drivers of the decline in the household saving rate. The consumer's ability to borrow is both enhanced by financial innovations which allow consumers purchase a house with lower downpayment requirements and financial instruments which help homeowners access home equity. In contrast, Parker (1999) is more conservative about this mechanism and argues that the channel alone can only

[11]Personal savings is defined as one minus the ratio of personal outlays to disposable income. Personal outlays is the sum of the consumption expenditures plus interest paid by persons and personal transfer payments abroad.

explain a limited part of the increase in consumption share, if any. This paper tries to quantitatively evaluate the impact of financial liberalization on the decline in the saving rate and the consumption boom.

2.3 Simplified Economy

In the previous section, we outlined the institutional origins of traditional mortgage loans. In particular, we showed that traditional mortgage loans imply that homeowners are forced to accumulate equity as the loan matures ("forced saving"). Moreover, under the traditional mortgage regime, homeowners cannot access the home equity they accumulated in the past ("lock-in effect"). By making use of refinancing opportunities, homeowners can avoid accumulating equity in the first place or they can tap the home equity they accumulated in the past easier.

In this section, we analyze to what extent refinancing opportunities change the saving choice of homeowners and affect their risk-sharing opportunities. We will demonstrate that precautionary saving under the traditional mortgage regime is higher than that under the refinancing regime. Intuitively, this is because the features of the traditional mortgage contract amplify the impact of earnings uncertainty. We show, however, that, despite the fact that households accumulate more savings for consumption smoothing purposes in the traditional mortgage regime, households enjoy better risk-sharing in the refinancing regime.

Our analysis in this section is based on a simplistic life-cycle model which permits an analytical solution. In the next section, we present a quantitative model that incorporates more realistic life-cycle features and institutional details. That model, however, does not allow for an analytical solution. With the help of numerical methods, we will show that the main conclusions that we derive from this simple model still hold, and more importantly, they are quantitatively relevant.

2.3.1 Environment

We consider a simple life cycle economy, which is populated by a continuum of ex-ante identical households. Household take the interest rate, r, as given (partial equilibrium).

Each household lives for three periods, $t = 1, 2, 3$. The household receives labor income in the first two periods and pension benefits in the last period. Labor income in the first period, $y_1 = 1$, is deterministic. Income in the second period is uncertain and given by $y_2 = 1 \pm \epsilon$, where positive ($+\epsilon$) and negative ($-\epsilon$) shocks occur with the same probability, 0.5. Moreover, shocks are uncorrelated across individuals. Retirement is denoted by y_r, that is, $y_3 = y_r$.

Households receive period utility, $u(c_t, h_t)$, from consuming housing services h_t and nonhousing consumption c_t. We impose the restrictions $c_t > 0$ and $h_t > 0$.

Utility is separable across time. Total lifetime utility is thus given by

$$U = E_0 \sum_{t=1}^{3} \beta^{t-1} u(c_t, h_t).$$

We further assume that period utility is separable across good, i.e., $u(c_t, h_t) = u(c_t) + u(h_t)$. We assume households are risk-averse and prudent, namely, $u'(.) > 0$,$u''(.) < 0$ and $u'''(.) > 0$. For simplicity, we assume the interest rate $r = 0$ and the discount factor $\beta = 1$.

In this simple economy, we highlight the impact of the traditional mortgage loan structure by making the following assumptions. Households are required to be homeowners. At the beginning of their life cycle, they buy a house of size $\bar{h}$ through a mortgage loan. Households are not allowed to adjust the size of their housing size over the life cycle. We further assume that the housing stock does not depreciate and can be fully collateralized. Note, that these assumptions are relaxed in the quantitative model. [12]

2.3.2 Mortgage Loans

A house of size h is financed by a collateralized mortgage loan of size d. When households enter the economy, the mortgage loan is fully collateralized, and the initial mortgage loan is $d_1 = h^*$.

We consider the following two different mortgage regimes:

1. Traditional Mortgage: The bank requires repayment of the mortgage during the first period, and households cannot access their home equity afterwards: $d_t = 0$ if $t \geq 2$. In the following, we will label this regime as "no refinancing (NR) ".

2. Refinancing: Households can choose the payment stream, hence $0 \leq d_t \leq h^*$. We will also label this regime as "refinancing (R) ".

Regime 1 characterizes the payment requirement of the traditional mortgage loan. Regime 2 instead allows households to choose the payment stream of the mortgage debt. In extreme, households can choose not to repay their mortgage debt until the last period.

[12] In quantitative models, households are free to choose their housing consumption (i.e., the exact size of h). There will be transaction costs, so that changing owner-occupied housing is costly. Moreover, households can also decide whether they want to obtain their housing services from owner-occupied housing or by renting. And also the housing stock can only be fully collateralized. We will also relax the assumption regarding the separability of preferences across housing and nonhousing consumption.

Since un-collateralized borrowing is not allowed, households accumulate financial assets, $a \geq 0$, for consumption smoothing purpose. For simplicity, we define *net financial assets* $\widetilde{a}_t \equiv a_t - d_t$. We assume that initially, financial wealth is zero, i.e. $a_1 = 0$ and $\widetilde{a}_1 = -d_1$.

2.3.3 The Household's Problem

We now state the household's maximization problem backwards at each period of the life cycle.

Period 3: We start with the last period (Period 3). The optimization problem reads as follows:

$$\max_{c_3} \ u\left(c_3\right) + u\left(h^*\right),$$

$$c_3 + h^* = \widetilde{a}_3 + y_r + h^*,$$

where c_3 is the household's consumption at Period 3. Notice that in the traditional mortgage case, $\widetilde{a_3}^{NR}$ gives both the net and the total financial assets ($\widetilde{a_3}^{NR} = a_3^{NR}$), because $d_3 = 0$. In the refinancing case instead, $\widetilde{a_3}^{R}$ might be negative, as long as $\widetilde{a_3}^{R} \geq -h^*$.

Period 2: In Period 2, household's problem reads,

$$\max_{c_2} \ u\left(c_2\right) + u\left(h^*\right) + u\left(c_3\right) + u\left(h^*\right),$$

$$c_{2,i} + h^* = \widetilde{a}_2 + y_{2,i} - \widetilde{a}_3 + h^*.$$

The second period's income is uncertain, as income shocks can be good ($i = g$ and $y_{2g} = 1 + \epsilon$) or bad ($i = b$ and $y_{2b} = 1 - \epsilon$). The two mortgage regimes imply different constraints for net financial wealth. Under the traditional mortgage regime where homeowners cannot access their home equity, $\widetilde{a_3}^{NR} \geq 0$. In contrast, under the refinancing regime, $\widetilde{a_3}^{R} \geq -h^*$.

Period 1: The household's optimization problem in the first period reads as follows:

$$\max_{c_1} \ u\left(c_1\right) + u\left(h^*\right) + E\left(u\left(c_2\right) + u\left(h^*\right) + u\left(c_3\right) + u\left(h^*\right)\right).$$

such that

$$c_1 + h^* = y_1 - \widetilde{a}_2 + h^* + \widetilde{a}_1.$$

Notice that for simplicity, we assume that y_1 is deterministic. Similarly, the traditional mortgage regime requires $\widetilde{a_2}^{NR} \geq 0$, whereas in the refinancing regime, $\widetilde{a_2}^{R} \geq -h^*$. Recall that the initial holding of financial wealth is zero, i.e. $a_1 = 0$ or $\widetilde{a_1} = -h^*$ in both regimes.

2.3.4 Results

We need to make an additional assumption on housing size and income profile to make this problem interesting.

Assumption 1. $y_1 > h^* > y_{2b} + y_1 - y_r.$

This assumption states that the size of the initial mortgage is large relative to the lifetime income that a homeowner receives if the second period's income realization turns out to be bad. And it is smaller than the income in the first period. This assumption also implies $y_r > y_{2b}$.

With assumption 1, we can show the following result:

Lemma 1. In the traditional mortgage regime, the upper bound on household's saving at the end of period 1 is $\widetilde{a_2}^{NR} < y_r - y_{2b}$.

In other words, this Lemma gives $\widetilde{a_2}^{NR} + y_{2b} < y_r$. It means that in the traditional mortgage regime, if the income realization turns out to be bad, households have incentive to borrow against future income.

Result 1. Homeowners in the traditional mortgage regime will have net financial assets that exceed those of homeowners in the refinancing regime at period 2, $\widetilde{a_2}^{NR} > \widetilde{a_2}^{R}$.

That is, the net financial assets of homeowners at the beginning of the second period are higher in the traditional mortgage regime as compared to the refinancing regime.

This is what we call the "forced-saving effect"of traditional mortgage loans. It is important to notice that it hinges on the Lemma (1). The fact that homeowners in the traditional mortgage regime have an incentive to borrow but cannot cause an excess amount of savings in the first period with respect to the refinancing regime.

With Lemma (1) and Result (1), we show the "lock-in effect". Precisely, since borrowing is not possible at Period 2, optimal consumption choices in Period 2 and 3 are

$$c_{2b}^{NR} = y_{2b} + \widetilde{a_2}^{NR},$$

and

$$c_3^{NR} = y_r,$$

In contrast, consumption choices under refinancing regime,

$$c_{2b}^{R} = c_{3}^{R} = \frac{y_{2b} + y_r + \widetilde{a_2}^{R}}{2}.$$

In the traditional regime, consumption in Period 2 and Period 3 do not coincide, if the income shock turns out to be bad. On the other hand, refinancing opportunities allow households to achieve a flat consumption profile, given the negative shock realized in Period 2.

It is interesting to note that although households in the traditional mortgage regime accumulate more net financial wealth than households in the refinancing regime, they do not achieve a smooth consumption profile between Period 2 and Period 3. In other words, households in the refinancing regime save less but the fact that they can access home equity in Period 2 allows them to achieve a smoother consumption stream.

We now show that our findings have important implications for the ability of homeowners to share income risk. We measure the degree of risk-sharing by the dispersion of consumption across the various income realizations in period 2. In our economy, because utility is concave, perfect risk-sharing would imply that consumption is equal across all states of the income process. That is, the higher the degree of risk-sharing, the lower the dispersion of consumption.[13]

The next result then suggests that risk-sharing is better in the refinancing regime.

Result 2. The dispersion of consumption in Period 2 is greater for homeowners in the traditional mortgage regime compared to the dispersion of consumption in the refinancing regime.

$$c_{2g}^{NR} - c_{2b}^{NR} > c_{2g}^{R} - c_{2b}^{R}.$$

This finding is very interesting. It implies that, despite the fact that homeowners in the traditional mortgage regime accumulate more financial assets than homeowners in the refinancing regime, they actually achieve less risk-sharing. Put differently, when financial markets are deregulated, we would expect to see a decline in saving and a decrease in consumption variability, which is quantitatively consistent with the trends in the U.S. economy. In the next section, we present a quantitative model that allows us to evaluate the quantitative importance of this effect.

[13] In the quantitative model that we present in the next section, we measure the dispersion of consumption by variance of the percentage deviation of consumption from its mean.

2.4 Quantitative Model

2.4.1 Households

We consider an economy with one unit mass of finite-lived households, aged from 0 to J. Households face mortality risk along the life span, and the conditional probability of surviving is $\psi_j \in (0,1)$. The newborn, therefore, survive until age j', with a probability of $\Pi_{j=0}^{j'} \psi_{j+1}$. The population size of households at age j is μ_j.[14]

Each household supplies inelastic labor, $\bar{l}$, each period to the firm in final production sector until retirement age, j^*. Age specific productivity is v_j, which is the mean log-normal income of the j-year old. Additionally households face uninsurable income risks when they work. Individual i's period-specific earning shock is e_{it} at period t. And the law of motion of the earning risk is standard AR(1): $e_{t+1} = \rho \cdot e_t + \varepsilon_t$, where $\rho \in (0,1)$ is the persistence and ε_t is the white noise with a standard deviation σ_e. A worker's gross labor income in period t and age j, is $w_t \cdot e_t \cdot v_j$, where w_t is the wage rate. Working households pay tax on their gross labor income, and the tax rate is τ_t. Retired households receive a pension benefit, p, from the government. Therefore, the labor or pension income for household i in period t is $y_{i,t}$, and

$$y_{i,t} = \begin{cases} (1-\tau) \cdot w_t \cdot e_{it} \cdot v_j \cdot \bar{l} & j < j^* \\ p & j \geq j^*. \end{cases}$$

Another source of income for households is the return of their financial assets, a. The interest rate in this economy is r_t at period t, and the gross return is $(1+r)$.

Households value the consumption of a nondurable goods and housing services that can be obtained on the rental market or through home ownership. Precisely, households derive period utility, $u(c, h^s)$, from two types of consumption: non-durable consumption goods, c, and housing services, h^s. Households can either consume housing services by rental a house (of size f) from the renting market, or housing service provided by their own housing stock, h,

$$h^s = z \cdot h + (1-z) \cdot f,$$

where z is an index function, taking value 1 and 0: $z = 1$ for households being homeowners and $z = 0$ for being renters. Moreover, lifetime utility is discounted every period at a rate of $\beta > 0$.

[14]We assume households do not have bequest motives and leave remaining net worth as "terminal consumption", e.g. funeral and medical expenses (Favilukis et al. (2010)).

The depreciation rate for homeowner's housing stock is δ_o. And we assume that the homeowners pay $\delta_o \cdot h$ for maintaining their homes[15]. The depreciation rate for renting a house is δ_r.

Households can buy and sell houses in the housing market with transaction costs, which are in proportion to the housing size(s) they sell and/or buy, $tr\,(h', h)$, and

$$tr\left(h', h\right) = (1 - I(h = h')) \cdot (\tau_s \cdot h + \tau_b \cdot h').$$

where τ_s and τ_b are the proportions of transaction fees to the housing values homeowners sell and buy respectively. $I(x)$ is an indicator function, which takes value one if the relation x is true, and zero otherwise.

2.4.2 Market Arrangement

There are no state-contingent markets for the household-specific shocks and the set of financial assets is exogenously determined. In particular, there is only one financial asset, a bond, which pays interest independent of the realizations of income shocks. Therefore, households cannot fully insure against their idiosyncratic risks. They can only self-insure by saving, using bonds.

2.4.3 Government

The government taxes labor income and redistributes the revenue to the retired. Specifically, the government transfers pension benefit, p , to each retiree, in every period. Labor taxes τ_t are adjusted such that the government's budget is balanced.

2.4.4 Firm in the Goods Sector

There is one single-good in this economy, which is produced by a representative firm in the goods sector. The production technology is standard Cobb-Douglas,

$$Y = K^{\alpha} \cdot L^{1-\alpha}$$

where K represents aggregate capital, L aggregate labor demand and Y the output in the goods sector. Aggregate capital depreciates at a rate of δ_k. In this economy, aggregate labor supply is exogenous and determined by the age-specific productivity and the idiosyncratic productivity of households in this whole economy. Since there is no aggregate risk, the aggregate labor supply is constant in this economy.

[15] In other words, we assume that homeowners have to maintain all the depreciated part of the housing stock they own.

2.4.5 Renting Market, Housing Market and Financial Sector

There is a competitive financial intermediary sector with a large number of risk neutral financial firms. Competition among those firms drives profit to zero in equilibrium.

Financial firms collect savings from both foreign and domestic households. We denote the aggregate deposits from domestic households A^d and the aggregate foreign deposits A^f.

Financial firms can transform households' financial assets into productive capital, residential capital and owner-occupied housing stock, on a one-to-one basis without any adjustment costs. Therefore, the relative price between productive capital and housing stock/residental capital, as well as the relative price between productive capital and consumption are always one. K is the capital rented to the representative firm that produces final goods. Residential housing, F, is the total supply in the renting market to the renters. Financial firms also construct owner-occupied homes and sell them to households. Households can also borrow from financial firms, using housing stock as collateral. In other words, financial firms can issue mortgages to homeowners.[16] We denote the aggregate mortgage loan D. Table 2.2 presents the balance sheet of this sector.

Table 2.2: Balance Sheet

Assets	Liabilities
Productive capital, K	Domestic deposits, A^d
Mortgage loans, D	Foreign deposits, A^f
Residential capital, F	

For simplicity, borrowing and lending rates are assumed to be the same, r. Therefore, net return to mortgage loan is r. The no-arbitrage condition implies that in equilibrium, the financial firms must be indifferent between renting productive capital to the firm, offering mortgage loans to homeowners and lending residential housing to renters. Therefore, the rental rate of capital is $r + \delta_k$, and renting price must be equal to the sum of the depreciation of renting and the interest rate, i.e., $r_f = r + \delta_r$.

Following Henderson and Ioannides (1983), we assume that $\delta_r > \delta_o$. This implies that in equilibrium, the price for renting is higher than the user cost of

[16]For example, a household who wants to buy a house of value h, pays h to a financial firm to purchase the house. The financial firm constructs a home of value h and sells it to the household. The household can also borrow from the financial firm to finance his purchase, using the housing stock as collateral. The loan from the financial firm to the household is the collateralized mortgage loan.

owner-occupied housing, $r + \delta_r > r + \delta_o$. This provides incentives for households to become homeowners. It is important to notice that without this incentive, no household would choose to become a homeowner in this model, since renting strictly dominates owning, ceteris paribus. This is because housing services acquired from the rental market do not involve frictions related to housing, such as transaction costs, downpayment requirements or mortgage payments. Our assumption that rental housing depreciates at a higher rate than owner-occupied housing can be justified with moral-hazard on the side of the tenant.

2.4.6 Mortgage Structures

This subsection characterizes mortgage payment structures and the household's problems accordingly. To provide a stylized contrast between mortgage market arrangements prevailing in the U.S. economy before and after deregulation, we focus on two polar cases: 1) households purchase home through traditional, fixed-term level payment mortgage contracts but cannot refinance; and 2) households can refinance without any cost.

Our modeling strategy is as follows. In both cases, purchasing a house requires a downpayment, which is proportional to the home's value. This downpayment ratio is denoted by ϕ_1. The difference between the two polar cases lies in the flexibility of the two types of payment schedules. Without refinancing opportunity, households have to accumulate home equity as the payment schedule specifies. In contrast, with refinancing opportunities, households can "rewrite"the mortgage contract to choose the payment stream over the contract period. Moreover they can also borrow from the financial intermediary sector, using housing stock they accumulate as collateral.

Traditional Mortgage: Fixed-Term, Level-Payment Mortgage

Homeowners in this model acquire their home equity with a fixed-payment or fixed-rate mortgage. To model this mortgage contract, we closely follow Chambers et al. (2009a, 2009b). We use similar techniques to formalize the constant payment schedule, which is an important feature of the traditional mortgage regime.

The fixed-term, level payment mortgage loan requires homeowners make *constant* mortgage payments over the length of the mortgage contract. It implies an increasing amortization schedule of the principal and a decreasing schedule for interest payments. Specifically, the initial debt level is d_0, when acquiring a new house, $d_0 = (1 - \phi_1) \cdot h'$, where h' is the size of the purchased house. The constant payment each period is $m = \lambda \cdot d_0$, where λ is a constant number which balances the principal and interest payment schedules. In other words, it is pinned down by the mortgage rate and the length of maturity, or $\lambda = r \cdot \left[1 - (1 + r)^{-T}\right]^{-1}$, where r is mortgage rate and the T is the length of the mortgage contract. Therefore the housing debt evolves as follows:

$$d_t = d_{t-1} \cdot (1 + r) - m.$$

As stressed before, there is no refinancing opportunity in this type of contract, and homeowners constantly reduce the housing debt level until they own all of the home equity, unless they sell the house before the contract ends.

Homeowners could sell their houses or upgrade or downgrade their housing stock holdings. If homeowners sell their houses and become renters, part of the housing value is used for repaying the remaining debt, and homeowners keep the home equity they have accumulated in the past. Similarly, if the homeowners upgrade or downgrade their housing stock holdings, the initial contract is cleared and they must sign a new contract with new debt holding.

To characterize the household's decision in this environment, we introduce another state variable, residual time s, which represents the length of maturity (the number of periods) left before the contract ends. On the one hand, the residual time, s, represents the debt position information of the homeowner. On the other hand, it helps to distinguish different types of households in this economy. For example, if $s = 0$ (residual time is zero) and $h = 0$ (housing stock holding is zero) for households, it implies they are renters in this economy. If $s = 0$ (residual time is zero) and $h > 0$ (housing stock holding is positive), it implies they are homeowners who own the house without any debt. Moreover, If $s > 0$ (residual time is positive) and $h > 0$ (housing stock holding is positive), it implies they are homeowners who own the part of the house that has a positive debt position.

Generally, in this economy, households, either renters or homeowners, choose consumption of nondurable goods and asset holding. If households choose to become homeowners (or upgrade/downgrade their housing stock), they also decide on the size of the rental unit. If they choose to become renters (or continue to be renters), they have to decide on the size of the rental unit as well. If homeowners stay in their own house, without changing housing size, they consume housing services generated by the housing stock they own and do not need to choose housing size.

Imagine that the homeowners in this economy are hit by negative income shocks. They could be able to use liquid financial assets - deposits - to smooth their consumption. For homeowners with low liquid assets, they can sell or downgrade their housing stock with a positive transaction cost. In other words, they cannot utilize the home equity accumulated in their homes, unless they sell their houses. Because of the transaction cost, there is an "inaction zone" where homeowners with low liquid assets do not adjust their housing stock, even after being hit by a negative income shock. Even in that case, they must pay the mortgage loan as scheduled, namely the committed expenditures. In this case, they have to adjust their consumption to a lower level.

Household's problem Households choose (z', c, a', h', f) to maximize,

$$V_j\left(e,a,h,s\right)=\max_{c,h^s,a',s',z'}\left\{u\left(c,h^s\right)+\psi_{j+1}\cdot\beta\cdot E_{e,e'}V_{j+1}\left(e',a',h',s'\right)\right\},$$

where $c>0, a'\geqslant 0, h'\geqslant \underline{\mathrm{h}}, f\geqslant 0, s'\geqslant 0, z'=1$ or 0. Note that we assume the minimum housing size in this economy is $\underline{\mathrm{h}}$. In other words, homeowners can choose housing size between $[\underline{\mathrm{h}},\infty]$.

The law of motion for s, especially needs elaboration. For households who were renters yesterday and become homeowners today, $s=0$ and $s'=T$. For households who were renters yesterday and are still renters today, $s=0$ and $s'=0$. For households who were homeowners yesterday and become renters today, $s\geqslant 0$ and $s'=0$. For households who were homeowners yesterday and upgrade or downgrade their housing today, $s\geqslant 0$ and $s'=T$.

The choice is subject to the following budget constraint,

$$\begin{aligned}&c+a'+\left(1-z'\right)\cdot r_f\cdot f+tr\left(h',h\right)+\lambda\cdot\left(1-\phi_1\right)\cdot h\cdot I\left(s>0\right)\cdot I\left(h=h'\right)\\&+\phi_1\cdot h'\cdot\left(z\cdot\left(1-I\left(h=h'\right)\right)+\left(1-z\right)\right)+\delta_o\cdot h\cdot z\cdot I\left(h=h'\right)+\delta_o\cdot h'\cdot z'\cdot\left(1-I\left(h=h'\right)\right)\\&=w\cdot e\cdot v_j\cdot\left(1-I\left(j\geqslant j^*\right)\right)+p\cdot I\left(j\geqslant j^*\right)+\left(1+r\right)\cdot a+z\cdot\left(h-d_{T-s}\right)\cdot\left(1-I\left(h=h'\right)\right).\end{aligned}$$

where indicator function $I\left(x\right)$ takes value one if the relation x is true, and zero otherwise. Left-hand side of the equation gives the spending decisions households make, while the right hand side gives household's resources. All the households have to decide on consumption c, and asset holding, a'. If households choose to become renters for this period, $z'=0$, they have to choose the size of the rental unit, f. And if homeowners choose to upgrade or downgrade their housing stock or become renters, they decide on housing size, h' or renting size, f. A transaction cost, $tr\left(h',h\right)$, has to be paid by households, who make housing transactions. If they are homeowners at the beginning of this period and decide not to move, $I\left(h=h'\right)=1$, they make mortgage payments, $\lambda\cdot\left(1-\phi_1\right)\cdot h$, conditional on their mortgage contract has not been finished, $I\left(s>0\right)=1$. Otherwise, homeowners pay zero in mortgage payments. Homeowners at the beginning of this period who want to upgrade and downgrade their housing, pay a new downpayment for the new house they buy during this period. The same is true for renters at the beginning of this period who choose to become homeowners. Homeowners who do not want to move, must maintain their house by paying $\delta_o\cdot h$. Movers and new home owners, must maintain the newly purchased house, $\delta_o\cdot h'$.

The right-hand side of the budget constraint describes household resources. The working cohorts of households receive labor income of $w\cdot e\cdot v_j$, and the retired cohorts receive pension benefits of p. All of them receive returns on the asset holding $\left(1+r\right)\cdot a$, unless the asset position is zero. For homeowners who want to move, they need to sell their house and clear up the housing debt, if any. The rest become resources for them to use, $h-d_{T-s}$. The household problem can be decomposed into several distinct situations.

Renters who continue to be renters Suppose that the household is a renter at the beginning of the period and continues to be a renter during this period again. The housing stock holding is simply zero both at the beginning and at the end of the period, that is $h = 0$ and $h' = 0$. The renter only needs to decide on renting size f and there is no mortgage-related expenditures. Denote the labor income or pension income as $y_{e,j} = w \cdot e \cdot v_j \cdot (1 - I(j \geqslant j^*)) + p \cdot I(j \geqslant j^*)$. The recursive form of the decision problem is stated as follows:

$$V_j(e, a, 0, 0) = \max_{c,f,a'} \left\{ u(c, f) + \psi_{j+1} \cdot \beta \cdot E_{e,e'} V_{j+1}\left(e', a', 0, 0\right) \right\}$$

$$s.t.$$

$$c + a' + r_f \cdot f = y_{e,j} + (1 + r) \cdot a.$$

Homeowners who are becoming renters Suppose that the household is a homeowner at the beginning of the period and chooses to be a renter during this period. The housing stock is positive both at the beginning of the period and zero at the end of the period, that is $h > 0$ and $h' = 0$. The household only needs to decide on renting size f. The difference from the last case is that the homeowner needs to sell the house he owns and repays the debt he owes to the bank, $h - d_{T-s}$. At the meantime a transaction cost is incurred, $tr(0, h)$. The recursive form of the decision problem is stated as follows:

$$V_j(e, a, h, s) = \max_{c,f,a'} \left\{ u(c, f) + \psi_{j+1} \cdot \beta \cdot E_{e,e'} V_{j+1}\left(e', a', 0, 0\right) \right\}.$$

$$s.t.$$

$$c + a' + r_f \cdot f + tr(0, h) = y_{e,j} + (1 + r) \cdot a + h - d_{T-s}.$$

Homeowners who choose to stay Suppose that the household is a homeowner at the beginning of the period and chooses to stay in his own house during this period. The housing stock is positive both at the beginning of the period and remains the same at the end of the period, that is $h = h'$. The household does not need to decide on the housing size. The home owner needs to maintain the house and pay $\delta_o \cdot h$. If the standard contract is not finished yet, or $s > 0$, he also has to make the mortgage payment, $\lambda \cdot (1 - \phi_1) \cdot h$. If he owes the entire home equity or $s = 0$, he need not pay the mortgage. The recursive form of the decision problem is stated as follows:

$$V_j(e, a, h, s) = \max_{c,a'} \left\{ u(c, h) + \psi_{j+1} \cdot \beta \cdot E_{e,e'} V_{j+1}\left(e', a', h, \max(s - 1, 0)\right) \right\}$$

$$s.t.$$

$$c + a' + \lambda \cdot (1 - \phi_1) \cdot h \cdot I(s > 0) + \delta_o \cdot h = y_{e,j} + (1 + r) \cdot a.$$

Homeowners who upgrade or downgrade Suppose that the household is a homeowner at the beginning of the period and chooses to upgrade or downgrade his housing stock during this period. The housing stock is positive both at the beginning and at the end of the period, although they are not the same, that is $h \neq h'$. The household needs to decide on the housing size, h'. The homeowner first needs to sell the house he owns and repays the debt he owes to the bank. The remaining equity is $h - d_{T-s}$. He has to start a new mortgage contract to buy a new house. Therefore, the state variable which keeps track of the number of periods left before the mortgage contract finishes, is reset to T. He has to pay a downpayment for the new house, $\phi_1 \cdot h'$, the transaction cost incurred, $tr(h', h)$, and the maintenance fee, $\delta_o \cdot h'$. The recursive form of the decision problem is stated as follows:

$$V_j(e, a, h, s) = \max_{c,h',a'} \left\{ u\left(c, h'\right) + \psi_{j+1} \cdot \beta \cdot E_{e,e'} V_{j+1}\left(e', a', h', T\right) \right\}$$

$$s.t.$$

$$c + a' + tr\left(h', h\right) + \phi_1 \cdot h' + \delta_o \cdot h' = y_{e,j} + (1 + r) \cdot a + h - d_{T-s}.$$

Renters who are becoming homeowners Suppose that the household is a renter at the beginning of the period and chooses to be a homeowner during this period. The housing stock at the beginning of this period is zero and positive at the end of this period, that is $h = 0$ and $h' > 0$,respectively. He needs to decide on the housing size. He has to make a downpayment for the new house, $\phi_1 \cdot h'$, the transaction cost incurred, $tr(h', 0)$ and the maintenance fee, $\delta_o \cdot h'$. The recursive form of the decision problem is stated as follows:

$$V_j(e, a, 0, 0) = \max_{c,h',a'} \left\{ u\left(c, h'\right) + \psi_{j+1} \cdot \beta \cdot E_{e,e'} V_{j+1}\left(e', a', h', T\right) \right\}.$$

$$s.t.$$

$$c + a' + tr\left(h', 0\right) + \phi_1 \cdot h' + \delta_o \cdot h' = y_{e,j} + (1 + r) \cdot a.$$

Flexible Refinancing

This subsection details the other polar case, that is, homeowners have options to refinance. Refinancing in this model is defined broadly. Except for changing downpayment ratio (or the initial home equity holding), all the refinancing activities, including taking additional mortgage loans and negotiating payment schedules are considered. This definition implies homeowners would be able to alter the monthly payments owed on the loan, or by altering the loan's term of maturity.. One could think of a case in which homeowners can sign a new contract at the end of every period (periodically refinancing). Moreover, they can also borrow from the financial intermediary by using their home equity as collateral. In essence, homeowners can access collateral lending, subject to the constraint that mortgage debt does not to exceed a certain limit. Downpayment is considered as the minimum home equity, which must be owned by the homeowner. We adopt the assumption that total borrowing cannot exceed a fraction, $1 - \phi_1$, of home value. This assumption has been widely used in the housing literature (for example, Diaz and Luengo-Prado(2008)). Therefore, a homeowner's debt evolves as follows:

$$d' \leqslant (1 - \phi_1) \cdot h'.$$

Imagine that the homeowners in this economy are hit by negative income shocks. They could use liquid financial assets - deposit - to smooth their consumption. For homeowners with low liquid assets, they could also make use of the home equity they have accumulated, without selling their house. It is the refinancing channel of consumption smoothing.

Household's problem Households choose (z', c, a', h', f) to maximize,

$$V_j(e, a, h, d) = \max_{c, h^s, a', z'} \left\{ u(c, h^s) + \psi_{j+1} \cdot \beta \cdot E_{e,e'} V_{j+1}\left(e', a', h', d'\right) \right\}$$

where $c > 0$, $a' \geqslant 0, h' \geqslant \underline{\mathrm{h}}, f \geqslant 0, z' = 1$ or 0. Technically, comparing with the previous case, we can see that the state variable s is not being useful for homeowners to make their decision, since households could refinance any time when they need to. The budget constraint for households is the following:

$$\begin{aligned} & c + a' + (r+1) \cdot d \cdot I\left(h = h'\right) + \left(1 - z'\right) \cdot r_f \cdot f + tr\left(h', h\right) \\ & + h' \cdot \left(z \cdot \left(1 - I\left(h = h'\right)\right) + (1 - z)\right) + \delta_o \cdot \left[h \cdot z \cdot I\left(h = h'\right) + \cdot h' \cdot z' \cdot \left(1 - I\left(h = h'\right)\right)\right] \\ & = w \cdot e \cdot v_j \cdot \left(1 - I\left(j > j^*\right)\right) + p \cdot I\left(j > j^*\right) + r \cdot a + (h \cdot z - d) \cdot \left(1 - I\left(h = h'\right)\right) + d'. \end{aligned}$$

The left-hand side of equation provides spending decisions of households. All the households have to decide on consumption c, and asset holding, a'. If households choose to become renters for this period, $z' = 0$, they have to choose the size of the rental unit, f. And if homeowners choose to upgrade or downgrade their housing stock or become renters, they decide on housing size, h', or renting size, f. Households, that make housing transactions, must pay a transaction cost, $tr\left(h', h\right)$. If they are homeowners at the beginning of this period and decide not to move, $I\left(h = h'\right) = 1$, they service mortgage debt they hold by paying $r \cdot d$. Homeowners at the beginning of this period who want to upgrade or downgrade their housing pay h' for the new house. This is the same for renters at the beginning of this period who choose to become homeowners. Homeowners who do not want to move must maintain their house by paying $\delta_o \cdot h$. Movers and new home owners must maintain the newly purchased house, $\delta_o \cdot h'$.

The right-hand side of the budget constraint provides the household's resources. The working cohorts of households receive labor income $w \cdot e \cdot v_j$, and the retired cohorts receive pension benefit of p. All of them receive returns on the asset holding $(1 + r) \cdot a$, unless the asset position is zero. Homeowners who want to move must sell their house and clear up the remaining debt. The rest becomes resources for them to use, $h - d$. Homeowners are also free to choose a new debt level, d'. Several distinct situations can be also laid out as follows.

Renters who continue to be renters Suppose that the household is a renter at the beginning of the period and continues to be a renter during this period as well. The housing stock is simply zero both at the beginning and at the end of the period, that is $h = 0$ and $h' = 0$. The renter only needs to decide on renting size f. Technically, the form of mortgage contract does not affect this type of households. The recursive form of the decision problem is stated as follows:

$$V_j\left(e, a, 0, 0\right) = \max_{c,f,a'} \left\{u\left(c, f\right) + \psi_{j+1} \cdot \beta \cdot E_{e,e'} V_{j+1}\left(e', a', 0, 0\right)\right\}$$

$$s.t.$$

$$c + a' + r_f \cdot f = y_{e,j} + (1 + r) \cdot a.$$

Homeowners who are becoming renters Suppose that the household is a homeowner at the beginning of the period and chooses to be a renter during this period. The housing stock is positive both at the beginning of the period and zero at the end of the period, that is $h > 0$ and $h' = 0$. The household only needs to decide on renting size f. The difference from the last case is that the homeowner needs to sell the house he owns and repays the debt he owes to the bank. The remaining equity is $h - d$. The new debt level is zero, since he chooses to be a renter this period. At the meanwhile, a transaction cost is incurred, $tr\left(0, h\right)$. It It appears that the form of mortgage contract does not take effect on households in this situation. However, implicitly, homeowners might be less likely to choose to be renters in the current market arrangement than homeowners who have no refinancing opportunities in the previous case, given reasonable negative income shocks. They could choose to stay and refinance, unless they are close to the borrowing limit. The recursive form of the decision problem is stated as follows:

$$V_j\left(e,a,h,d\right)=\max_{c,f,a'}\left\{u\left(c,f\right)+\psi_{j+1}\cdot\beta\cdot E_{e,e'}V_{j+1}\left(e',a',0,0\right)\right\}$$

$$s.t.$$

$$c+a'+r_f\cdot f+tr\left(0,h\right)=y_{e,j}+\left(1+r\right)\cdot a+h-d.$$

Homeowners who choose to stay Suppose that the household is a homeowner at the beginning of the period and chooses to stay in his own house during this period. The housing stock is positive both at the beginning of the period and remains the same at the end of the period, that is $h = h'$. The household does not need to decide on the housing size. He needs to maintain the house and pay, $\delta_o \cdot h$. Unlike the traditional mortgage case, where he makes a constant mortgage payment and cannot change the debt holding, there is no committed payment to fulfill and he can adjust the debt level. However, he has to pay the interest on the existing debt. The recursive form of the decision problem is stated as follows:

$$V_j\left(e,a,h,d\right)=\max_{c,a'}\left\{u\left(c,h\right)+\psi_{j+1}\cdot\beta\cdot E_{e,e'}V_{j+1}\left(e',a',h,d'\right)\right\}$$

$$s.t.$$

$$c+a'+\left(1+r\right)\cdot d+\delta_o\cdot h=y_{e,j}+r\cdot a+d',$$

$$d'\leqslant\left(1-\phi_1\right)\cdot h.$$

Homeowners who upgrade or downgrade Suppose that the household is a homeowner at the beginning of the period and chooses to upgrade or downgrade his housing stock during this period. The housing stock is positive both at the beginning and at the end of the period, although they are not the same, that is $h \neq h'$. The household needs to decide on the housing size, h'. The homeowner first needs to sell the house he owns and repay the debt he owes to the bank. The remaining equity is $h - d$. He has to pay for the new house. He could choose to borrow against home equity, and he decides on the new debt level, d'. A transaction cost incurred and must be paid, $tr\left(h', h\right)$. The recursive form of the decision problem is stated as follows:

$$V_j\left(e,a,h,d\right)=\max_{c,h',a'}\left\{u\left(c,h'\right)+\psi_{j+1}\cdot\beta\cdot E_{e,e'}V_{j+1}\left(e',a',h',d'\right)\right\}$$

$$s.t.$$

$$c+a'+tr\left(h',h\right)+h'+\delta_o\cdot h'=y_{e,j}+\left(1+r\right)\cdot a+h-d+d',$$

$$d'\leqslant\left(1-\phi_1\right)\cdot h'.$$

Renters who are becoming homeowners Suppose that the household is a renter at the beginning of the period and chooses to be a homeowner during this period. The housing stock at the beginning of this period is zero and positive at the end of this period, that is $h=0$ and $h'>0$. The household needs to decide on the housing size. He has to pay for the new house. He could choose to borrow against home equity, and he decides on the new debt level, d'. A transaction cost incurred has to be paid, $tr\left(h',0\right)$. The recursive form of the decision problem is stated as follows:

$$V_j\left(e,a,0,0\right)=\max_{c,h',a'}\left\{u\left(c,h'\right)+\psi_{j+1}\cdot\beta\cdot E_{e,e'}V_{j+1}\left(e',a',h',d'\right)\right\}$$

$$s.t.$$

$$c+a'+tr\left(h',0\right)+h'+\delta_o\cdot h'=y_{e,j}+\left(1+r\right)\cdot a+d',$$

$$d'\leqslant\left(1-\phi_1\right)\cdot h'.$$

2.4.7 Stationary Equilibrium

We focus on an open-economy stationary equilibrium in which the interest rate, r, is determined exogenously by the international market.[17] The representative firm chooses the optimal factor inputs in production then determines the wage rate per efficiency unit, w. An exogenous interest rate is particular important and attractive for this model: we want to single out the net effect of contract regimes, and it is extremely important to fix the interest rate in both economies. The interest rate affects mortgage debt payment, asset return, as well as household's

[17] A similar approach has been used by Fernandez-Villaverde and Krueger (2010), Kaplan (2007) and Heathcote, Storesletten, and Violante (2008).

inter-temporal choice. In a close economy equilibrium, however, the interest rate is endogenously higher in the refinancing regime, since households accumulate less precautionary savings, which boosts the interest rate. Moreover, one important feature of the U.S. economy is that domestic investment is financed by resources from abroad. Therefore it is more suitable to adopt the open-economy equilibrium.

The state of an individual is fully characterized by a vector of state variables, including idiosyncratic labor productivity, liquid asset holdings, housing stock, mortgage status or debt information and household age. For notational efficiency, we denote the state vector $x = \{e, a, h, s, j\}$ and the state space $X = \{E \times [0, \overline{a}] \times [\underline{h}, \overline{h}] \times S \times J\}$, where $\overline{a}$ and $\overline{h}$ are the upper bounds for financial assets and housing stock in this economy.[18] And the distribution of the households over the state space X is Ω.

1. The representative firm in the goods sector takes interest rate, r, as given, and chooses an optimal factor inputs ratio of $\frac{K}{L}$:

$$r = \alpha \cdot \left(\frac{K}{L}\right)^{\alpha-1} - \delta_k.$$

   And the wage rate, w, is implied:

$$w = (1-\alpha) \cdot \left(\frac{K}{L}\right)^{\alpha}.$$

2. The domestic labor markets clear: given wage rate w, a firm's labor demand equals the aggregation of individual labor supply, $L = \int_X e \cdot v \cdot \bar{l} \; d\Omega$.
3. Given factor prices, interest rate r and wage w, the value function and decision rules are solved for the household optimization problem, namely, $V(x)$, $a'(x)$, $h'(x)$, $f(x)$,$s'(x)$ and $c(x)$.
4. The government runs balanced budget, $T = P$, where $T = \tau \cdot w \cdot L$ is the total labor income tax from working cohorts, and $P = p \cdot \sum_{j=j^*}^{J} \mu_j$ is the total pension payment to retired cohorts. The implied tax rate is $\tau = \frac{P}{w \cdot L}$.
5. Financial market clears, $A^d = \int_X a'(x) \; d\Omega$.
6. Renting market clears, $F = \int_X f(x) \; d\Omega$.
7. Housing market clears, $H = \int_X h'(x) \; d\Omega$.
8. Financial intermediary firm maximizes profit and earns zero profit.

[18]The variables $\overline{a}$ and $\overline{h}$ are so large that households do not accumulate financial assets and housing stock more than $\overline{a}$ and $\overline{h}$, given the exogenous income process.

9. Domestic goods market clears, namely,

$$C + \delta_o \cdot H + \delta_k \cdot K + \delta_r \cdot F + \Upsilon + NX = K^{\alpha} L^{1-\alpha}.$$

   where C, H, K, F, Υ and NX are the aggregate nonhousing consumption, aggregate housing stock, aggregate physical capital, residential housing, aggregate transaction cost and current account in this economy.

10. World asset market clears, which requires the change in net foreign asset position equals the current account, namely,

$$r \cdot (D + K + F - A^d) = NX.$$

   where D is domestic aggregate mortgage debt.

11. The distribution, Ω, over the whole state space, i.e., age, financial assets, housing stock, earning shocks, and mortgage status (or debt) is invariant.

2.5 Quantitative Analysis

In this section, we evaluate how financial liberalization affects aggregate trends, such as the saving rate/consumption share, debt holding and the homeownership rate. We use the model with traditional mortgage loans as a benchmark, which approximates the pre-deregulation economy in early 1980. To analyze the impact of financial liberalization, we conduct the following experiment:

Experiment 1: Compute the model with traditional mortgage loans with a high downpayment ratio.

Experiment 2: Compute the model with refinancing opportunities with a high downpayment ratio.

Experiment 3: Compute the model with refinancing opportunities with a low downpayment ratio.

Experiment 4: Compute the model with traditional mortgage loans with a low downpayment ratio.

The differences between Experiments 1 and 3 gives the full impact of financial deregulation. To analyze the impact of refinancing opportunities, we compare results from Experiments 1 and 2. To analyze the role that low downpayment ratio plays during financial liberalization, we compare results between Experiments 2 and 3. We also conduct a counter factual Experiment 4, where a lower downpayment ratio is allowed but refinancing opportunities are not available. The comparison between Experiments 1 and 4, allows us to identify how the lowering downpayment ratio alone could affect the aggregate trends.

It is important to note that, one of our contributions is that results from our experiments can separate the effects of lowering downpayment requirements from the effects of refinancing. In reality, both increasing availability of refinancing and decreasing downpayment requirements happened at the same time during financial liberalization. Previous studies usually regard lower downpayment requirements as an increase in access to housing equity, and use this change alone to approximate the impact of financial liberalization. However, as we highlight in previous text, given the standard payment schedule (or the first mortgage regime in this paper), low downpayment requirements do not necessarily imply more access to home equity at all. As acknowledged by Díaz and Luengo-Prado (2009), previous literature cannot "disentangle one from the other"(page 21).

2.5.1 Calibration Strategy

We choose to calibrate the economy with traditional mortgage loans to match important features of the pre-deregulation U.S. economy in the early 1980s. Calibration of the benchmark economy is the in the following order: demographics and timing, preferences, the earnings process, the technology, and housing and renting markets. The parameters are summarized in Table 2.3.

Demographics and Timing To ease the computational burdens, one model period is five years.[19] Households are born at age 20, and die at the maximum age of 80. The retirement age is 60. Survival probabilities are taken from United Nations (2002), which provides the survival probability for the year of 2000.

Preferences We use the following standard Constant Relative Risk Aversion (CRRA) utility function. The Cobb-Douglas aggregator between housing consumption and non-durable consumption is used.

$$u(c, h^s) = (c^{\gamma} \cdot h^{s1-\gamma})^{1-\sigma}/(1-\sigma)$$

Housing services provided by owner-occupied housing are assumed to be same as the housing stock owned by the owners. The coefficient of risk aversion, σ, is set to 2, which is commonly used in the literature. The weight households put on nonhousing consumption, γ, is calibrated to match the ratio of housing services to

[19]We translate all the parameters at annual frequency into the ones for a model period of five years. All the parameters reported are annual data.

nonhousing consumption, which was 0.23 in the early 1980s.[20] The annual discount factor, $\beta = 0.9575$, is calibrated to match the nonhousing wealth to income ratio, 2.5, (See Kaplan and Violante 2009).

Earnings Process There are a number of estimations of risky earning process in the literature. We choose to follow Díaz and Luengo-Prado (2009).[21] They estimate that the persistence and variance are $\rho = 0.9895$ and $\sigma_e = 0.0158$, respectively.[22] We approximate the AR(1) process by a 5-point Markov chain, using the procedures described in Tauchen (1986).

The mean efficiency index over a life cycle is taken from Hansen (1993) so that the model could replicate the cross-section age distribution of earnings of the U.S. economy. Notice that the age-specific productivity profile is hump-shaped and peaks at age 50. The household's inelastic labor supply is chosen to be $\bar{l} = 0.3$. The replacement ratio is the ratio of retirement pension income to last year's working cohorts' labor income. It is taken from Chambers et al. (2009a) and set at 0.3. The labor income tax rate is solved endogenously, consistent with the assumption of a balanced government budget.

Technology Similar to Fernandez-Villaverde and Krueger (2010), we fix the interest rate at 3% in both of the economies by choosing a capital level K. The wage rate for efficient labor unit is implied. The depreciation rate of capital, δ_k, is set at 0.08. The share of physical capital in output is $\alpha = 0.30$, which is standard in the literature.

Housing and Renting Markets We set the selling cost equal at 7 percent and buying cost at 2 percent of the housing values. They are consistent with Martin and Gruber (2004), who document buying and selling cost with CEX data. The downpayment ratio is set at 0.20 for the early 1980s case, which is close to the average downpayment reported by the Federal Housing Finance Board and similar to Campbell and Hercowitz (2009)'s estimation as well, see Table 2.1.

We calibrate renting depreciation rate, δ_r, the owning depreciation rate, δ_o, and the minimum housing size, $\underline{h}$, to jointly match the economy-wide homeownership rate, which was 64% in early 1980s, housing to physical capital stock ratio, 0.60, and the housing investment-stock ratio, 0.043.

[20] Housing services are defined as the value of housing consumption, that is, the sum of renting and owning housing stock evaluated by the renting price.

[21] Alternatively, we can choose to follow Storesletten, Telmer, and Yaron (2004), where earning is defined as wage earnings plus transfer. They also allow for transitory shocks and fixed effects. Therefore, their estimations of persistence and variance are $\rho = 0.9989$ and $\sigma_e = 0.0166$. Díaz and Luengo-Prado (2009)'s estimation is closer to our purpose. However, it is important to know that the difference in various estimations is small and they give very similar results.

[22] They find that this process can deliver a reasonable Gini index for earnings of regular households (as in the SCF-98), and a coefficient of variation (CV), which is very similar to the one in the SCF-98 for households outside the top 1 percent of the earnings distribution.

Post-Deregulation U.S. Economy The post-deregulation U.S. economy is characterized by the model with refinancing opportunities and lower downpayment ratios. We use exactly the same set of parameters for this economy, except that households in this economy can access refinancing opportunities and the downpayment requirement is lower. Table 2.1 shows that the average equity-value ratio for newly purchased homes in 1983 was 22.6%, and this ratio has decreased to 16.4% by 2001, Campbell and Hercowitz (2009). We thus experiment with a downpayment ratio of 15% for the post-deregulated economy.

Table 2.3: Baseline Parameters: Pre-Deregulation

Parameter	Economic interpretation	Value
	Demographics	
J	Maximum age	11
ψ	Mortality risk	United Nations (2002)
	Preferences	
σ	CRRA risk aversion	2.00
β	Annual discount rate	0.9575
γ	Nonhousing consumption weight	0.85
	Technology	
η	Capital share	0.30
δ_k	Capital depreciation rate	0.08
	Earning process	
ρ	Persistence	0.989
σ_e	Standard deviation	0.0158
υ_j	Efficiency index	Hansen(2003)
	Housing market	
ϕ_1	Downpayment ratio	0.20
τ_b	Percentage cost of buying	0.02
τ_s	Percentage cost of selling	0.07
δ_r	Renting depreciation rate	0.14
δ_o	Owning depreciation rate	0.043
$\underline{h}$	Minimum housing size	0.40

Source: Standard and Calibrated Parameters

2.5.2 Quantitative Results

Homeownership Rate and Debt Holding

Table 2.4: Aggregate Trends in the Model

Downpayment	Regime	Ownership	Debt Ratio	Cons Share
High	NR	64%	26%	92.6%
High	R	70%	47%	93.8%
Low	R	75%	56%	94.9%

Source: Numerical Experiments.

The homeownership rate is the percentage of homeowners in this economy. The homeownership rate is substantially higher in the case where refinancing is allowed. It goes up from 64% to 70%, given that we keep the downpayment ratio as high as 20%. Lowering the downpayment ratio further helps to increase the homeownership rate by 5 percentage points. Compared to data where the homeownership rate increases from 64% to 69%, the model over predicts the change.[23]

Without refinancing opportunities, homeowners who experience negative income shocks cannot make use of home equity they accumulate, unless they choose to sell or downgrade their housing stocks. Refinancing options allow homeowners to access their home equity without selling or downgrading their home. Therefore, there are two channels by which the homeownership rate can increase. Firstly, refinancing opportunities create fewer renters, given the same magnitude of negative income shocks. Secondly, anticipating that home equity can be easily accessed, households have more incentive to build up housing stock and start accumulating housing stock earlier. Compared with the previous literature which argues that demographic changes are important for the increase in homeownership rate (see Chambers et al. 2009a), we stress financial liberalization also plays an important role.

It should be noted that our model shows that allowing homeowners to access equity can induce more households to purchase homes. This effect plays a major role during financial liberalization. Lowering downpayment does help increase the homeownership rate but turns out to be relatively less important.

We observe that the debt to labor income ratio increases in the refinancing model, compared with the traditional mortgage model. The net mortgage debt to labor income ratio is only 26%, in the case of the traditional mortgage. And it

[23] Our model over-predicts changes in homeownership rate. It is largely due to the fact that our experiments assume an uniformed downpayment ratio and equal access to refinancing opportunities. In reality, some households may not be eligible for refinancing, and effective downpayment ratios also vary across groups with different eligibility.

increases to 47%, when we allow homeowners access to refinancing opportunities. Decreases in downpayment ratios drives this ratio further to 56%. In total, the two mechanisms combined can deliver an increase of 30 percentage points, roughly in line with the increase of 37 percentage points in the data. While we under-predict the absolute level of net mortgage debt, we successfully deliver the substantial change in debt levels.[24] We also observe that the new borrowers increased by 8 percentage points, from 18% in the pre-deregulation economy, up to 26% in the post-deregulation economy. It is also consistent with the increase of 7 percentage points in the data.

These results are intuitive. Firstly, they go hand in hand with the fact that the homeownership rate is higher. More households are willing to become homeowners and purchase homes. They borrow from the financial intermediary, using housing stock as a collateral. This results in a higher level of debt, given a certain labor income. Secondly, refinancing opportunities allow homeowners to raise more debt, when adverse income shocks occur. In the traditional mortgage environment, homeowners with very low liquid assets, they have to sell or downgrade their housing stock to insure themselves against adverse income shocks. In the case of refinancing, the homeowners can borrow against their home equity to smooth out consumption. Thirdly, lowered downpayment requirements allow homeowners to make use of an even larger share of the home equity, and the first two mechanisms are amplified. Comparing the two channels of financial liberalization, we also find those opportunities which allows homeowners to access home equity has a stronger effect than lowered downpayment requirement does.

Scoccianti (2009) also finds that a lower downpayment ratio does not generate substantial increase in net debt. He concludes that earning shocks must be more permanent over time so that more households are borrowing constrained and higher debt levels arise. However, Blundell, Pistaferri, and Preston (2008) find that the transitory component of earning risk in the labor income process has increased, and the permanent component has actually decreased. It seems difficult to reconcile Scoccianti (2009)'s conclusion to what the data suggest. We use a very standard, stochastic labor income processes for both the pre and post-deregulation economy. The experiments show that the net mortgage debt can increase substantially even without any increase in the persistence of income shocks. We stress that refinancing opportunity plays a key role in the rising indebtedness during financial liberalization.

[24] As it is well-known in the literature, the parameters used in our calibration of the earnings process are not capable of generating enough wealth inequality, compared to the data (see, for example, Castaneda et al. 2003). Clearly, this also implies too little debt in the aggregate, since the wealth-poor are too rich with respect to the data. Our calibration strategy is standard in the literature, which facilitates a comparison to the previous literature. We leave it to future work to re-calibrate the earnings process to match wealth inequality and debt holdings exactly.

The Consumption Boom and The Decline of Saving Rate

As subsection 2.2.5 showed, it is useful to use the consumption share of personal income to measure the trend of household saving rate.[25] We observe that the aggregate consumption share (the ratio of aggregate consumption to disposable income) does increase by 2.3 percentage points, from 92.6% to 94.9%. We find that allowing for refinancing options accounts for roughly half of the increase generated by the model, while the lower downpayment ratios accounts for the other half. It demonstrates that financial liberalization is responsible for roughly one-third of the consumption boom in the U.S. economy.

We find that precautionary savings is substantially lower in the post-deregulation economy. With refinancing, housing capital becomes a close substitute for the liquid assets (or financial saving), although still inferior, in terms of insurance against earning shocks. Because the housing stock becomes relatively more "liquid" and consuming owner-occupied housing is cheaper than renting, households choose to hold relatively more housing stock than households in the pre-deregulation economy. Meanwhile, they save less liquid financial assets out of precautionary motives.

Less precautionary savings release more resources for consumption purposes. Since owner-occupied housing also provides housing consumption services, households consume more housing services as well.

Feldstein (2008) points out that financial innovations and refinancing in the housing market help homeowners access home equity; therefore, they borrow more and save less. Our model shows that financial liberalization helps households build up more housing stock and save less in financial assets. In other words, they "save" more in housing assets and they consume more housing services.

Mendoza et al. (2007) argues that financial development in the U.S. economy helps to reduce precautionary saving. Therefore, part of the investment is financed by foreign capital inflow. In our model, we also observe both of them. We lend support to Mendoza et al. (2007) by showing that financial liberalization in the housing market is an important aspect of the financial development.

Downpayment Requirements: High vs. Low

In this subsection, we identify the impact that lowering downpayment requirements alone could have, on the aggregate trends. We thus conduct a counter-factual experiment, where the downpayment requirement is lower but refinancing is still not

[25] The aggregate consumption is defined by the sum of the nonhousing consumption and imputed housing consumption. Housing consumption is composed of renting service and housing service generated by owner-occupied housing. Following the "user cost "approach, it is evaluated by "owning price", namely the depreciation rate of owner-occupied housing plus interest rate. According to the NIPA definitions, however, the imputed value of owner-occupied housing is computed by using the "rental price". In our case, this is higher than the user-cost approach. This implies that the increase in consumption would be even larger if we followed the NIPA definition. Personal income is defined by the sum of domestic households' labor income and return to financial assets.

allowed. Interestingly, it turns out that the homeownership rate and debt holding do not increase. Similarly, consumption share does not increase either. These results are less surprising, because homeowners in this model are still required to build up home equity over time, despite the lowered downpayment requirements. They still cannot make use of the home equity they have accumulated in the past.

However, as we documented before, we observe an increase in the homeownership rate, debt holding and consumption share, when we decrease downpayment requirements in the model with refinancing. These results are also intuitive. Since homeowners in this model could make use of an even larger share of home equity, the effects of removing forced-home-equity-saving (or accessing to home equity) on the aggregate trends are amplified.

In summary, these results show that: 1) financial instruments, which allow homeowners to have better access to home equity are critical part of financially liberalization. And without refinancing opportunities, lowering downpayment ratios only affects the trends marginally at the best; 2) However, the lowered downpayment ratio amplifies the effect induced by the availability of refinancing opportunities.

The lessons learned from these experiments have strong policy implications: policies which encourage low downpayment ratios alone are not responsible for the substantial change in the aggregate trends, while policies that promote refinancing opportunity are critical.

Risk-Sharing

Our analytical model shows that allowing for refinancing options lowers precautionary saving and leads to more risk-sharing. Our results from the previous sections showed that the drop in precautionary saving is indeed substantial. We now analyze the impact of financial deregulation on risk-sharing. We measure risk-sharing as the variance of the individual (nonhousing) consumption share $C_i/\bar{C}$, where $\bar{C}$ is the average consumption. In an environment with perfect risk-sharing, their consumption across time and income states, the variance is zero or $C_i = \bar{C}$. We compute this measure for the whole population in both economies. Since the refinancing channel directly helps the working population who face labor income risks to smooth out consumption, we also compute the risk-sharing for the working population. The main results are summarized in the Table 2.5.

We find that for the whole population, the variance of individual consumption relative to the average consumption decreases from 0.617 in the traditional mortgage economy to 0.610 in the refinancing economy. If we lower the downpayment ratio, it drops further to 0.608. The total difference in the risk-sharing measures between the pre- and post-deregulated economies is 0.009. And the difference between working populations in these two types of economies is relatively larger, 0.015. In the traditional mortgage economy, the variance is 0.608. And in

Table 2.5: Risk-Sharing

Downpayment	Regime	Whole Pop	Working Pop
High	NR	0.617	0.608
High	R	0.610	0.594
Low	R	0.608	0.593

Source: Numerical Experiments.

the refinancing case, it drops to 0.594, and the lower downpayment ratio reduces the variance further to 0.593. This larger difference is expected, since refinancing helps the working population to insure against labor income risks, while the retired population in both types of economies do not face income risks.

It should be noted that the variance we compute is a measure of the overall risk-sharing effect of different insurance channels. Since, in the traditional mortgage case, the insurance channels are limited to self-insurance (for the whole population) and downgrading housing size (for the homeowners). In the refinancing case, the additional risk-sharing channel (using home equity) would allow households to rely less on the self-insurance. One evidence is that the aggregate precautionary savings in the refinancing case is substantially lower than the non-refinancing case. In other words, they substitute the home-equity risk-sharing channel for the standard precautionary-saving, self-insurance channel. Our results show that refinancing opportunities make households better off in terms of risk-sharing, despite the fact that they make less use of the self-insure mechanism. In other words, the difference in the risk-sharing measures of the two regimes under-predicts the effect of risk-sharing provided by the refinancing channel.

Our results contribute to the literature that measures risk-sharing opportunities in incomplete market models. In an important contribution, Krueger and Perri (2006) argue that allowing for more debt (through relaxing borrowing constraints) in an Aiyagari-type incomplete markets model results in less risk-sharing because debt is state-contingent. We show that this result does not hold in a life-cycle economy when the increase in debt is generated by relaxing payment requirements of mortgage loans.

2.6 Conclusion

In this paper, we evaluated the impact of deregulation of housing finance with a quantitative life-cycle model. Overall, our model can deliver a substantial increase in net mortgage debt, one third of the increase of total consumption share. The increase in homeownership rate is also consistent with the aggregate trend. Moreover, we can separate the effect of the two aspects of financial liberalization. We also show refinancing opportunities play a key role in financial liberalization.

There are several important aspects of the housing market which have been left out in this research. First of all, we assume that refinancing is costless. And we can consider that the exercise provides an upper bound for the effect of refinancing. This assumption also implies that different forms of refinancing have an equivalent impact on the aggregate economy. In reality, refinancing activities differ substantially in many respects, for example cost, timing and eligibility etc. Introducing different types of refinancing could help quantify the significance of each type of refinancing. Moreover, housing prices may have a substantial impact in risk-sharing and the household saving rate. Housing prices appreciation would induce even more nonhousing consumption and further reduce the saving rate, through the wealth effect. We acknowledge that they are important issues and leave them to future work.

Chapter 3

Why is the Correlation Between Savings and Investment So Low in Developing Countries?

Chapter Summary

Feldstein and Horioka (1980) argue that the high correlation between savings and investment observed in OECD countries implies that the capital is strongly immobile internationally. This paper revisits one related stylized fact: The correlation in question is actually lower for developing countries than for developed countries. Existing theories with "frictional market approach" seem to be at odds with this empirical pattern: If the developing countries are less integrated into the international markets, it would imply a higher correlation between savings and investment. This paper proposes an explanation based on the permanent income theory. A small open economy, which is hit by a transitory positive productivity shock, increases both savings and investment, so that they are positively correlated. Permanent shocks lead to the opposite scenario: savings and investment respond in different directions, which results in a negative correlation. As shown empirically by Aguiar and Gopinath (2007), permanent shocks dominate in the lower income emerging economies, while transitory shocks around stable trends dominate in industrial economies. Therefore, the correlation should be relatively lower in developing countries than in developed countries.

Key words: Capital Mobility, Trend Shocks, Developing Countries

JEL classification: E3, F3

3.1 Introduction

This paper revisits an interesting stylized fact that the correlation between savings and investment in developing countries is generally lower than that in developed countries. I show that one reason underlying this empirical pattern is that properties of productivity shock process in developing countries are different from their developed counterparts. The purpose of this paper is then to provide a quantitative assessment of this explanation.

The Feldstein-Horioka puzzle has been one of the most important issues in the field of international macroeconomics. Feldstein and Horioka (1980) argue that the correlation between savings and investment should be quite low, suppose that the international financial market is close to perfect or the capital mobility is high. However, their empirical results show that even for the developed OECD countries, domestic savings and investment move quite closely to each other. They regress domestic investment rates on domestic savings rates with data from 16 OECD countries, i.e., the regression is specified as

$$\left(\frac{I}{Y}\right)_i = \alpha_0 + \alpha_1 \left(\frac{S}{Y}\right)_i + \epsilon_i$$

in which $\left(\frac{I}{Y}\right)_i$ and $\left(\frac{S}{Y}\right)_i$ are average investment rate and savings rate for a country i in a given period. And ϵ is the error term. α_0 is the constant term, while α_1 is the coefficient in concern and often referred as "FH coefficient".

They find a positive and substantial FH coefficient, which is close to one. This evidence has been used to support the argument that international financial market is not perfect but frictional. Feldstein-Horioka puzzle stimulated a large amount literature which both empirically re-exam and theoretically explain this high correlation (see Coakley et al. (1998) and Tesar (1991) for review of this issue). Among others, Bai and Zhang (2009) is one of the most recent research, which aims at explaining this empirical pattern by financial frictions.[1]

However, another aspect of this regression analysis has not attracted as much attention as the Feldstein-Horioka puzzle. As shown in various studies, both *cross-section wise* and *time series wise*, not only the savings-investment correlations are high in general, but also they are even higher for industrial countries than for the developing countries.

For example, Frankel et al. (1988) is one of the cross-section studies, which exams the savings and investment behaviors in these two groups of countries in the period of 1960-84. And they document that the correlation is substantially weaker for developing countries than for developed countries. This finding is the same as in Summers (1989). Obstfeld and Rogoff (2001) also run the same regressions with a different dataset in a shorter time horizon. They find quite similar results: Country groups with higher income level tend to have higher FH coefficient.

[1] They confirm the puzzle by running the same regressions with a longer period and updated data (1960-2000).

Table 3.1: Related Evidence

Sample	1990-1997
Full Sample	0.41
	(0.008)
Countries with GNP/Cap > 1000	0.48
	(0.09)
Countries with GNP/Cap > 2000	0.70
	(0.09)
OECD Countries	0.60
	(0.09)

Source: Obstfeld and Rogoff (2001)

I use updated World development Indicators (WDI) data from 1960 to 2000 and run the same regressions with the whole sample, as well as with the sub-samples of advanced OECD and developing countries respectively. [2] I also run regressions for each sub-period with the whole sample and the two subs-samples. The full sample results are just similar to Feldstein and Horioka (1980) and Bai and Zhang (2009): the FH coefficient is indeed positive and significant.[3] Comparison between the advanced OECD and developing countries gives interesting results. The FH coefficient for developed countries is always higher than that for developing countries in any sub-periods. For the whole period, 1960-2005, the gap is 0.22. For the sub-periods of 1960-74 and 1974-2000, the gap is 0.41 and 0.10, respectively.

Table 3.2: Savings-Investment Correlations

Sample	1960-2000	1960-1974	1974-2000
Developing Countries	0.43	0.28	0.46
Advanced OECD	0.69	0.69	0.56
Full Sample	0.46	0.46	0.45
Feldstein-Horioka	n.a.	.89	n.a.

Source: WDI and Feldstein and Horioka (1980)

Following the "time-series approach" first proposed by Obstfeld (1989), Mamingi (1997) estimates savings and investment correlations for 58 developing countries,

[2]The choice of sample countries follows Bai and Zhang (2009). The advanced OECD countries are the same as Feldstein and Horioka(1980). The rest of the sample is labeled as developing countries.

[3]The difference in the empirical results between Feldstein and Horioka (1980) and Bai and Zhang (2009) comes from data sources they use as well as the changes in systems of national accounts (SNA) (See Bai and Zhang 2009 for details).

using annual time-series data for the period from 1970 to 1990. He evaluates the degree of capital mobility in the Feldstein-Horioka sense for these countries. And his finding suggests that for the majority of developing countries, the correlations between savings and investment are indeed lower than those for developed countries.

This paper shows that the savings-investment correlation is affected by the relative importance of the permanent (or transitory) component in the productivity shock process. The developing countries experience distinct productivity shock processes from their developed counterparts, and therefore, their savings and investment behaviors are different as well.

Suppose there are two types of productivity shocks in a small open economy. Firstly, consider the scenario that the small open economy is hit by a transitory positive technological innovation ("cycle shock"). This results in an immediate increase in income. However, since the shock is transitory, it has little effect on the household's permanent income. Therefore, consumption responds to the shock by less than the increase in income, so that domestic savings rises. At the same time, the positive productivity shock shifts up the marginal productivity of capital. A standard no-arbitrage argument implies that the economy shifts resources to capital formation. Resources are directed in capital and investment booms. Therefore, it results in a positive correlation between investment and savings.

Secondly, consider a shock to the growth trend ("trend shock"). Current positive shock implies higher income today and even higher income in the future. Consumption responds more than income and savings have to decrease to smooth consumption. Similar to the previous case, the positive shock to productivity raises investment. And therefore, the correlation between savings and investment will be negative before the economy adjusts back to the steady state.

Thirdly, empirically, one of the remarkable and fundamental features of lower income economies is that non-stationary shocks (or shocks to trend growth) are the primary sources of their fluctuations. In contrast, stationary shocks (or transitory fluctuations around a stable trend) are driving the business cycles for industrial economies (Aguiar and Gopinath 2007).[4] If a developing economy is hit by a productivity shock, it is more likely to be a trend shock, then savings and investment will be less positively correlated. In contrast, if a developed economy is hit by a productivity shock, it is more likely to be transitory, then savings and investment will be more positively correlated. Therefore, the lower savings-investment correlation in developing countries emerges as the consequence of their different underlying shock properties from developed countries.

This empirical pattern has long been neglected, when efforts are made to explain the Feldstein-Horioka puzzle. Previous research on Feldstein-Horioka puzzle could be grouped as "frictional market approach"and "frictionless market approach". The first line of research follows Feldstein-Horioka's hypothesis that

[4] They identify the information on the persistence of productivity process with consumption and net exports behaviors.

financial frictions are important. For example, Bai and Zhang (2009) include enforcement constraint into an one-bond small open economy. They find endogenous borrowing constraint is more stringent for poor countries with low capital stock and poor countries can borrow much less than developed countries. Endogenous borrowing constraint limits the capital flow from rich countries to poor countries, when poor countries receive positive productivity shocks. This effect is driving the high correlation between savings and investment in the world. However, in their model, poorer economies are more closed and co-movement between savings and investment is stronger. Their mechanism seems to be at odds with the lower correlations in developing countries.

Other frictions have also been proposed to solve the puzzle. Obstfeld and Rogoff (2001) show that costs in international goods market trade drive a wedge between the effective domestic real interest rate and the world real interest rate. This wedge or "kink" in the inter-temporal budget constraint can induce the home country to behave in a fairly autarkic manner. However, the trade costs are bigger, at least no smaller in developing countries than developed countries. Generally, if we follow Feldstein-Horioka's conjecture, i.e., financial frictions (or other type frictions) are to be blamed for the high correlation, it implies that the correlation is larger for developing countries than for developed countries.

Another line of research tries to understand the high correlation between savings and investment with perfect mobility of capital cross countries. Those models rely on persistent exogenous shocks to generate positive correlations between savings and investment. Both Finn (1990) and Mendoza (1991) argue that the correlation between savings and investment in a small open economy can be quite high, even if the capital market is perfect mobile. Any kind of savings-investment correlation can be created, depending on the underlying stochastic process. Therefore, theoretically, positive correlations between savings and investment should not be interpreted as a low degree of capital mobility. Similarly, this paper also shows that the key determinant is the persistence of productivity shocks. However, this paper focus on the difference between developing and developed countries, instead of industrial countries alone. I calibrate the model such that it can match a number of moments of the average statistics in developing and developed countries. Therefore, I can quantitatively evaluate how the relative importance of the permanent (transitory) component affects the correlation between savings and investment.

The rest of the paper is organized as follows: In the next section, I will set up a model based on Aguiar and Gopinath (2007), i.e., a simple small open economy with both transitory and permanent productivity shocks. The model is solved with Uhlig's numerical package (Uhlig 1995). Section 3.3 presents the quantitative analysis. Both impulse response and numerical analyses are included. And the section 3.4 concludes. The Appendix C.1 shows the details of analytical solution, while Appendix C.2 and C.3 provides the computational details.

3.2 The Model

This section lays out a simple small open-economy model, by closely following Aguiar and Gopinath (2007). Compared with the standard small open-economy RBC model with persistent shocks, the only deviation is to incorporate non-stationary shocks to productivity innovation process.

3.2.1 Environment

Household The economy is populated by a one unit mass of households with standard Cobb-Douglas utility. The representative household enjoys both consumption and leisure. The period utility at period t reads,

$$u_t = \frac{\left(C_t^{\gamma}\left(1-L_t\right)^{1-\gamma}\right)^{1-\sigma}}{1-\sigma},$$

where γ is the relative weight the household puts on consumption, while σ measures the risk aversion. L_t is the labor supply and therefore $N_t = 1 - L_t$ is the leisure consumed at period t. The representative household supplies labor and rents physical capital he owns, K_t, to firms in the goods sector, given the prevailing factor prices, wage rate, i.e., w_t and interest rate, r_t.

The representative household can borrow and lend in the international financial market. There is only one risk-free and one-period bond available for households to trade. The debt level at period t is B_t. Following both Aguiar and Gopinath (2007) and Schmitt-Grohe and Uribe (2003), it is assumed that the interest rate is increasing in the aggregate level of foreign debt.[5] Precisely, the risk premium increases in the amount of foreign debt. The interest is the sum of international interest rate r^* and the risk premium. Specifically,

$$r_t = r^* + \psi \cdot \left[e^{\frac{B_{t+1}-B^*}{\Gamma_t}} - 1\right], \tag{3.1}$$

where B^* is the steady state level of debt holding and ψ measures the sensitivity of interest rate to the changes in debt levels. This specification implies that $r_t = r^*$ at the steady state.

Therefore, the representative household makes his decisions, subject to the following resource constraint,

[5] As Schmitt-Grohe and Uribe (2003) points out, it is a well-known issue that the dynamics of standard small open economy is not well behaved, since it critically depends on the initial condition, i.e., net foreign assets position. To maintain the stationarity, additional assumptions have to be imposed. One of the popular approaches is to assume that the interest rate faced by domestic households is increasing in the level of outstanding debt.

$$C_t + I_t + \varPhi(K_t, K_{t+1}) \cdot K_t = Y_t + (q_t \cdot B_{t+1} - B_t)$$

where $\varPhi(K_t, K_{t+1})$ is an adjustment cost function, I_t the investment and $q_t = \frac{1}{1+r_t}$ the price of bond at period t. At period t, the household receives total income of Y_t. And he also pays back the outstanding debt, B_t, and raises new debt, B_{t+1}. Therefore, $q_t \cdot B_{t+1} - B_t$ represents the resources borrowed from international market for the purpose of consumption and domestic capital formation.

The household consumes C_t and invests I_t to adjust his capital holding from K_t to K_{t+1}. Let δ denote the depreciation rate of capital, and the stock of capital evolves according to

$$K_{t+1} = I_t - (1 - \delta) \cdot K_t$$

The adjustment is assumed to be costly.[6] And the cost function is chosen to be the following quadratic form,

$$\varPhi(K_t, K_{t+1}) = \frac{\phi}{2} \cdot \left(\frac{K_{t+1}}{K_t} - e^{\mu_g} \right)^2$$

where ϕ captures the importance of the adjustment cost. Moreover, the constant term in the adjustment cost function, e^{μ_g}, guarantees that adjustment costs are zero on the long-run non-stochastic growth path.[7]

The household's problem is to maximize the life time utility, subject to the period budget constraint.

Firm On the production side, there is a representative firm that produces one-good by employing labor, L_t, and capital, K_t, from the household sector in competitive factor markets. It also sells its output in a competitive goods market. Production function is also standard Cobb-Douglas,

$$Y_t = A_t \cdot K_t^{1-\alpha} \cdot L_t^{\alpha}, \tag{3.2}$$

where α is the labor's share of output in this economy and A_t is the productivity at period t, which the firm takes as given.

Productivity process is characterized by $A_t = e^{z_t} \cdot \Gamma_t^{\alpha}$. The first component aims to capture the standard productivity shock process ("cycle shocks") , in which z_t follows a standard AR(1) process,

[6] In small open economy models, it is standard to include the adjustment costs, so that it avoids excessive investment volatility in response to changes in the differential between domestic and international interest rates.

[7] Additional restriction on the adjustment cost function has to be imposed to ensure that the adjustment costs are zero in the non-stochastic steady state and the domestic interest equals the marginal product of capital net of depreciation.

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z, \tag{3.3}$$

where $|\rho_z| < 1$ and ε_t^z is iid with zero mean and standard deviation of σ_z. The second component, Γ_t, gives the stochastic trend in this model, which is assumed to be the cumulative product of "growth shocks" in the past, g_s, where $s \leq t$,

$$\Gamma_t = e^{g_t} \cdot \Gamma_{t-1} = \prod_{s=0}^{t} e^{g_s},$$

The growth shocks also follow an AR(1) process,

$$g_t = (1 - \rho_g) \cdot \mu_g + \rho_g \cdot g_{t-1} + \varepsilon_t^g, \tag{3.4}$$

where ε_t^g is an iid shock with zero mean and standard deviation of σ_g and $|\rho_g| < 1$. Moreover, μ_g gives the long run mean growth rate of productivity. Since the growth shocks are positively correlated and are cumulative over time, the growth trend in this model is stochastic and non-stationary.[8]

The firm's problem is relatively simple. It maximizes its profit at each period. The profit function at period t is defined by π_t,

$$\pi_t = A_t \cdot K_t^{1-\alpha} \cdot L_t^{\alpha} - w_t \cdot L_t - (r_t + \delta) \cdot K_t,$$

3.2.2 Recursive Formulation

The problem can be de-trended with the trend productivity up to the current period, Γ. The system is normalized with the following "change in variable" techniques,

$$x' = \frac{X'}{\Gamma},$$

where x' represents the de-trended version of variable X'. Formally, the de-trended version of the household problem can be restated in the following recursive formulation,

$$V(k, b, z, g) = \max_{c,l,k',b'} \frac{\left(c^{\gamma}(1-l)^{1-\gamma}\right)^{1-\sigma}}{1-\sigma} + \beta \cdot e^{g \cdot \gamma \cdot (1-\sigma)} E\left[V\left(k', b', z', g'\right)|z, g\right]$$

[8]The key difference between this model and the standard RBC model in small open economy setup is the additional non-stationary productivity shock. This assumption is a critical to Aguiar and Gopinath (2007)'s hypothesis and empirical estimations. They demonstrate that the observed economic activities at business cycle frequencies are driven by the shocks to stochastic trend in lower income emerging economies. In contrast, developed economies have rather stable trends and business cycles are mostly driven by transitory shocks.

The value function of the representative agent is a function of capital of his own, k, debt holding, b, and shock realizations of the two types, z and g.[9] The right-hand side of the Bellman equation consists of the utility derived from current consumption and leisure, as well as the discounted expected continuation value, which is conditional on both z and g.[10]

The representative household chooses consumption, leisure, capital and debt holding optimally. The representative household's choice is subject to the normalized version of resource constraint,

$$c + k'e^g = y + (1-\delta)\,k - \frac{\phi}{2}\left(\frac{k'}{k}e^g - e^{\mu_g}\right)^2 k - b + qb'e^g \tag{3.5}$$

Given the recursive formulation described above, the definition of a competitive equilibrium is stated as follows.[11]

1. Value function $V(\cdot)$ and policy functions $c(\cdot)$, $l_s(\cdot)$, $k_s'(\cdot)$ and $b'(\cdot)$ solve the household's maximization problem, taking factor prices given, namely, wage rate, w, and interest rate, r.
2. The representative firm maximizes profit π each period. And the factor prices are competitively determined.
3. The asset market clears so that $k_s'(k,b,z,g) = k_d$.
4. The labor market clears so that $l_s(k,b,z,g) = l_d$.
5. The goods market clears, which implies that Equation (3.5) holds.[12]

The optimal choices of the representative household are characterized by the following first order conditions,

$$\left[c^\gamma (1-l)^{(1-\gamma)}\right]^{-\sigma} \gamma c^{\gamma-1}(1-l)^{(1-\gamma)} = \lambda, \tag{3.6}$$

$$\frac{c}{(1-l)} = \frac{\gamma}{(1-\gamma)} y_L, \tag{3.7}$$

[9]It is also standard to assume that $\beta \cdot e^{\mu_g \cdot \gamma \cdot (1-\sigma)} \cdot (1+r^*) = e^{\mu_g}$. This assumption is the counterpart of the assumption $\beta \cdot (1+r^*) = 1$ in standard small open economy without trend shocks (see Schmitt-Grohe and Uribe 2003, for example). Note that if the long run growth rate $\mu_g = 0$, this condition reduces to the standard case.

[10]Although labor supply does not need to be trended, I replace L by l to make the notation consistent throughout the formulation.

[11]The subscript (s) on k_s and l_s indicates "supply "from households. And the subscript (d) on k_d and l_d indicates "demand ".

[12]By Walras's law, the fact that international asset market is cleared is implied.

$$\widehat{\beta} \cdot E\lambda' = q \cdot \lambda, \tag{3.8}$$

$$\widehat{\beta} \cdot E\lambda' \cdot \left[y_{k'} + (1-\delta) + \frac{\phi}{2} \left[\left(\frac{k''}{k'} e^{g} \right)^2 - (e^{\mu_g})^2 \right] \right] = \lambda \cdot \left[\phi \left(\frac{k'}{k} e^{g} + e^{\mu_g} \right)^2 + 1 \right], \tag{3.9}$$

where $\widehat{\beta} = \beta \cdot e^{g \cdot \gamma \cdot (1-\sigma) - g}$.[13] Some remarks are in order. Equation (3.6) shows that the co-state variable, λ, equals to the marginal utility of consumption. Equation (3.7) is the intra-temporal condition which ensures that the household chooses labor and consumption optimally. Equation (3.8) is the standard pricing formula for the bond, except that the discount factor is adjusted by the trend. The right-hand side of Equation (3.9) gives the marginal cost of increasing one unit of capital in current period, while the left-hand side gives the discounted and expected benefit of doing so. Note that if we assume that adjustment cost is irrelevant, $\phi = 0$, Equation (3.9) becomes the standard inter-temporal condition. Moreover, if we assume $\mu_g = g = 0$, the household's problem reduces to the standard small open-economy real business cycle model (see Schmitt-Grohe and Uribe 2003).

3.3 Quantitative Analysis

This section presents quantitative analysis of this model. In subsection 3.3.1, I first describe the basic solution algorithm I use for solving this model. To facilitate the exposition of intuitions in the model, I conduct the impulse response analysis in subsection 3.3.2. I describe the calibration strategy and present the numerical results from simulations in subsection 3.3.3 and 3.3.4, respectively.

3.3.1 Numerical Solution

The model does not admit any closed-form solutions and needs to be solved numerically. The model is solved numerically by following the method proposed by Uhlig (1995), which has been widely used to solve non-linear dynamic discrete time stochastic models. In essence, it is an Euler-equation based approach and requires a log-linear approximation of the system around the steady state. It also imposes the linear form of recursive law of motion around the steady state, so that the method of undetermined coefficient can be applied to solve the linearized equation system. The following steps are taken to implement the algorithm for solution and simulation.

[13]Please refer to the Appendix C.1 for detailed solution.

Firstly, I defined the exogenous state vector, $Z = [z, g]^T$, endogenous state vector, $S = [k, b]^T$, as well as the choice vector $M = [y, c, n, l, nx, s]^T$, where y represents output, n leisure, l labor, nx net export and s savings.[14]

Secondly, the equation system Υ that characterizes the problem, comprises of the equilibrium relationships between state variables and choice variables, as well as the laws of motion for state variables. Related equations are Equations (3.2), (3.3),(3.4), (3.1), (3.5), (3.7),(3.9) and (3.8). Moreover we also need the following identities, (3.10), (3.12) and (3.11), to complete the system,

$$n \equiv 1 - l, \tag{3.10}$$

$$s \equiv y - c, \tag{3.11}$$

$$nx \equiv y - X - c. \tag{3.12}$$

Equations (3.10), (3.11) and (3.12) describe the definitions of leisure, savings, and net export, respectively. Moreover, X in Equation (3.12) denotes the gross investment and $X = k'e^g - (1-\delta)\,k + \frac{\phi}{2}\left(\frac{k'}{k}e^g - e^{\mu_g}\right)^2 k$.

Thirdly, to obtain the steady state information, it is necessary to impose the conditions that $S' = S$, $M' = M$ and $Z = Z^* = [0, \mu_g]$ (see Appendix C.2 for details).[15] Moreover, the equation system Υ needs to be log-linearized around the steady state, so that the system is approximately linear in the log-deviations from the steady state (see Appendix C.3 for details).

Fourthly, to solve the linearized version of system Υ, the law of motion of S' and M' need to be found. Usually, it is standard to postulate that the recursive law of motion is linear, and specifically, it is characterized by the following equation,

$$\begin{pmatrix} S' \\ M' \end{pmatrix} = P \cdot \begin{pmatrix} S \\ Z' \end{pmatrix} \tag{3.13}$$

where P is the coefficient matrix, which governs the dynamics of the system around the steady state.

Fifthly, in theory, method of undetermined coefficients can be used to find the coefficient matrix, P. In practice, this paper uses the "Uhlig package"(Uhlig 1995), which implements the undetermined coefficients method numerically.

Sixthly, since the coefficient matrix, P, can fully characterize the dynamics of the linearized system, impulse response analysis can be conducted without simulating the model. Suppose that the system is initially at the steady state before

[14]Some of the choice variables are redundant but this formulation helps to make the numerical analysis easier. X^T represents the transpose of the vector X.

[15]The equation system Υ is highly non-linear and no much information can be extracted from it, unless we switch to analyzing its dynamics around the steady state.

the Period 0. And exogenous shocks deviate from Z^* at the beginning of Period 0 and returns to Z^* from Period 1 on. The law of motion, Equation (3.13), allows one to trace out the trajectories of all the variables from Period 0, until the system come backs to the steady state.

Finally, simulations are conducted by using Equation (3.13). It is first necessary to draw a sequence of realizations of the exogenous shock vector from its distribution matrix. Starting with the steady state information of the state variables, the dynamics of endogenous state variables can be recursively computed. Similarly, choice variables can be also computed with information both on exogenous and endogenous states in each period.

3.3.2 Impulse Response Analysis

Preceding the numerical analysis, this subsection aims at providing intuitions on how investment and savings respond to trend and cycle shocks in this model. For this purpose, I present an analysis of impulse responses to the two types of shocks. Figure 3.1 shows the dynamics of investment and savings from an initial steady state after a one-percent positive transitory shock, that is, $\epsilon_1^z = 0.01$. Figure 3.2 displays the impulse responses of these two variables, after a one-percent positive growth shock, that is, $\epsilon_1^g = 0.01$.

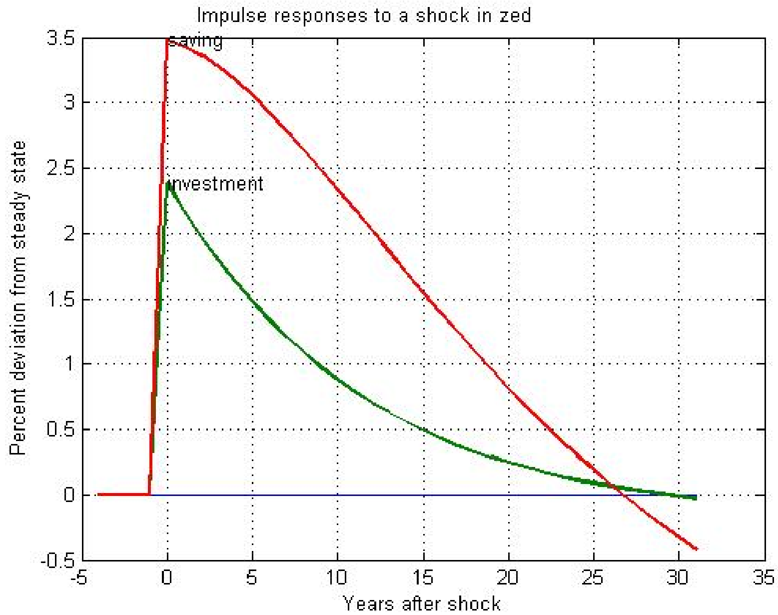
Initially, before Period zero, the economy is at the steady state level in both cases. Figure (3.1) and (3.2) shows that following one-percent of the two types of shocks, investments in the two distinct cases have similar responses: increase immediately from the steady state level, upon the point where the shocks realize, and adjust back to steady state level gradually. In contrast, the dynamics of savings is quite different in these two scenarios. On the one hand, following a transitory shock, savings increase first and adjusts back to the steady state sluggishly. On the other hand, savings decrease immediately when the shock realization is permanent and returns to the steady state in the following years.

The intuitions behind these different dynamics are based on the permanent income theory. Firstly, given a positive transitory shock to the productivity level, output increases. Since the shock is transitory, the household expects that the output will eventually decrease in the future. The households smooth the windfall gain over his life time. At the period of shock, consumption increases by less than the increase in output. Hence savings increase accordingly.

Secondly, consider a shock to the growth trend. The household observes that the economy is experiencing a period of high growth. Current positive shock implies higher output today and even higher output in the following periods. Consumption will respond more than the increase in output. It implies that savings have to decrease to smooth consumption.

Thirdly, the dynamics of investment in these two scenarios are pretty similar. In both cases, positive shocks enhance the productivity in the small open economy

Figure 3.1: Impulse Response to Cycle Shock



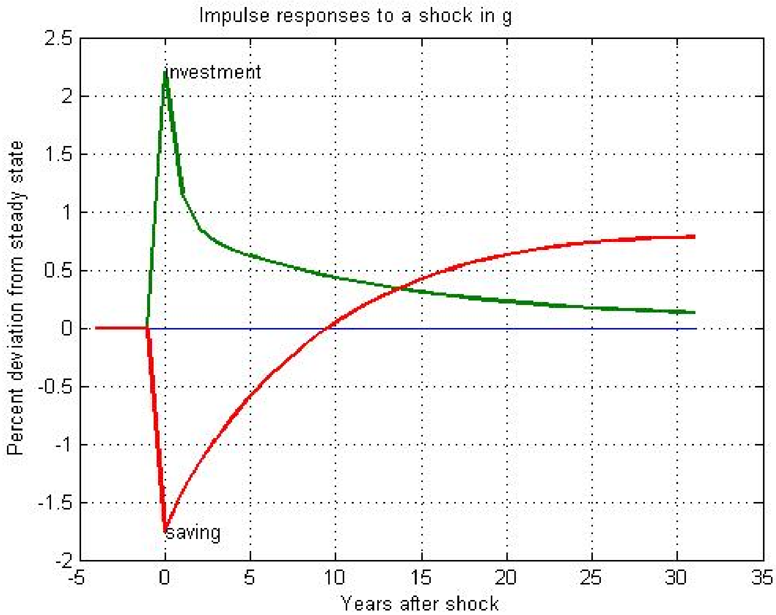
Note: The vertical axis gives the changes in *percent deviation* from the steady state for both investment and savings, after one-percent transitory shock.

and the marginal productivity of capital increases. Since the economy is small and the international interest rate is determined by the international interest rate and outstanding debt position. The initial real rate of return on bonds is relatively lower. A standard no-arbitrage argument implies that the economy shifts resources from bonds to capital formation, therefore investment increases upon the unexpected shocks. The only difference between effects of the different types of shock is that the investment boom lasts longer in the case of permanent shocks.

In summary, given a transitory shock, both savings and investment move in the same direction. Or in other words, transitory shocks trigger a positive comovements between savings and investment, which results in a positive correlation between these two variables. In contrast, savings and investment respond to the permanent shock in different directions, which results in a negative correlation.

In this model, therefore, the relative importance of permanent shocks in productivity innovation process matters for the correlation between savings and investment. If the permanent (transitory) shocks account for larger component the shock process, the correlation will be relatively lower (higher). In the following subsections, I evaluate quantitatively how the distinct shock processes in developing and developed countries can help to explain the different savings-investment behaviors in these two types of economies.

Figure 3.2: Impulse Response to Trend Shock



Note: The vertical axis gives the changes in *percent deviation* from the steady state for both investment and savings, after one-percent permanent shock.

3.3.3 Calibration Strategy

In this subsection, I briefly describe the calibration strategy for the numerical experiments. Throughout the experiments, I assume that the two types of economies are otherwise identical, except the shock processes they experience. I further assume that the persistences of cycle and trend shocks and the long run average growth rate, ρ_z, ρ_g and μ_g, are shared by both developing and developed countries. I calibrate ρ_g, so that the correlation of consumption, and output, in developed economy can be close to the data. I also choose to set $\rho_z = 0.95$, which is pretty standard for the stationary persistent shock.[16] Therefore, the pair of trend and cycle shocks variances distinguish lower income developing countries from the developed counterparts. I calibrate σ^z and σ^g for both developing and developed countries, so that the model can match the standard deviation of output, $\sigma(Y)$, and the autocorrelation of output, $\rho(Y)$, in both types of countries.

In the case of developing countries, both of the calibrated variances of permanent and transitory shocks are higher than their developed counterparts. More-

[16]These empirical moments are taken from (Aguiar and Gopinath 2007). However, my calibration strategy is different from their estimation process. They structurally estimate shock processes for only Canada and Mexico. I instead choose to match a set of average moments of developing and developed.

Table 3.3: Calibration Targets

Type	Moment	Value
Developed countries	$\sigma(Y)$, standard deviation	1.34
Developing countries	$\sigma(Y)$, standard deviation	2.74
Developed countries	$\rho(Y)$, autocorrelation	0.75
Developing countries	$\rho(Y)$, autocorrelation	0.76
Developed countries	$\rho(C, Y)$, correlation	0.66

Note: Average values of the selected moments for developing developed countries. Y and C are the output and consumption, respectively.

over, the ratio of the variance of permanent shocks to the variance of transitory shocks is higher, which is consistent with the fact that non-stationary component is relatively more important in developing countries.

The remaining parameters characterize the household's preferences and the firm's technology in this model. They are parameterized by using standard data in the RBC literature. The risk aversion parameter, σ, is chosen to be 2 and the weight on consumption is $\gamma = 0.36$. The quarterly discount rate β is standard from the literature, 0.98. The capital share, α, is set at 0.32, which is also common in RBC models. I choose the quarterly depreciation rate, δ, to be 0.05. I fix the adjustment cost parameter, ψ, at 2.5 for the benchmark case. Table 3.4 describes the benchmark parameters used in the model and summarizes the values.

3.3.4 Results

In this subsection, I present numerical results from simulations. I calculate the correlation between savings and investment by experimenting different sets of parameters and intend to investigate how the correlation between savings and investment responds to the relative importance of permanent (or transitory) component. I also study how the adjustment costs are responsible for the correlation by experimenting different adjustment cost parameters. Main results are summarized in Table 3.5 and 3.6.

I first experiment with the calibrated shock processes for developing and developed countries. I find that the correlation between savings and investment in the experiment for developing countries is 0.17, substantially lower than its counterpart in the case of developed countries, 0.37. It confirms the intuitions derived from the impulse response analysis. The permanent shocks trigger the opposite responses of savings and investment. And when they dominate in the productivity shock process of developing countries, the savings and investment is relatively less correlated than the developed economy case, where transitory shocks are relatively more important.

Table 3.4: Baseline Parameters

Parameter	Economic interpretation	Value
σ	CRRA risk aversion	2.0
β	Quarterly discount rate	0.98
η	Capital share	0.32
δ	Depreciation rate	0.05
ψ	Adjustment cost	2.5
ρ_z	Cycle shock persistence	0.95
ρ_g	Trend shock persistence	0.40
σ_z	Standard deviation of cycle shocks	1.25/0.68
σ_g	Standard deviation of trend shocks	2.28/0.85
μ_g	Long-run average growth rate	1.06

Source: Standard and calibrated parameters. Values before (behind) slash are the averages for the developing (developed) countries

Table 3.5: Benchmark Results

$\phi = 2.5$	$\sigma_z = 1.25$	$\sigma_z = 0.68$
$\sigma_g = 2.28$	0.17	-0.06
$\sigma_g = 0.85$	0.60	0.37

Source: Simulation

To understand the role of each shock component, I also carry out two counterfactual experiments. Firstly, in the developing countries case, I keep the variance of permanent shocks unchanged and decrease the variance of transitory shocks from 1.25 to 0.68, so that the transitory component plays an even less important role in the productivity shock process. It is shown that the correlation decreases substantially from 0.17 to -0.06. In other words, greater permanent component can even drive the correlation to be even negative. Secondly, I keep the variance of transitory shocks unchanged and decrease the variance of permanent shocks from 2.28 to 0.85. It is observed that the correlation in question increases from 0.17 to 0.60. It is also intuitive: Given the transitory shocks play greater role in this productivity process, the correlation is more positively correlated.

In summary, recessions (or booms) in the two types of economies are different objects. If recessions (or booms) hit industrial economies, they tend to be more transitory and return to the relatively stable trend in shorter periods. If recessions (or booms) hit lower income countries, they tend to be more permanent and last for longer periods. Therefore, households adjust their savings and investment behaviors differently.

Table 3.6: Low Adjustment Cost

$\phi = 1.0$	$\sigma_z = 1.25$	$\sigma_z = 0.68$
$\sigma_g = 2.28$	0.15	-0.08
$\sigma_g = 0.85$	0.50	0.30

Source: Simulation

In this model, it is straightforward to study how the adjustment costs affect the correlation between savings and investment. The high adjustment costs limit the flow of capital from and to the international markets. With low adjustment costs, the correlation between savings and investment will be even lower: since it is easier to adjust investment through the international market, the investment decision will be less constrained by the domestic savings. Table 3.6 presents the results from experiments where adjustment costs, ψ, is set at 1.0, a lower value than the benchmark case. Comparing with table (3.5), all the correlations computed from similar experiments are lower in Table 3.6.

3.4 Conclusion

This paper revisits one interesting stylized fact which has long been neglected in the field of international finance: not only the savings-investment correlations are high in general, but also they are even higher for industrial countries than for the developing countries. This seems to be at odds with the Feldstein-Horioka's hypothesis that international market frictions are driving the positive and high correlation between savings and investment. This paper proposes an explanation based on Aguiar and Gopinath (2007)'s new empirical findings: in lower income economies, permanent shocks are relatively more important in the productivity shock process, while in industrial economies transitory shocks dominates. This paper shows that the relative importance of permanent shocks matters for the correlation between savings and investment. If the permanent (transitory) shocks account for larger component the shock process, the correlation will be relatively lower (higher).

This research shows that developing countries adjust their savings and investment behavior quite differently from the developed countries, when they can be fully integrated into the international markets. One natural question should be addressed in future research: what are the potential gains from the access to the international markets for developing countries, given their underlying shock process are quite different from the developed counterparts? Are these gains bigger or smaller, comparing to the developed countries? And related to this question, what are the potential losses of being cut off from the international markets, given the random walk component accounts for a large share of the Solow residual variation? I leave these questions for the future research.

Appendices

Appendix A

Appendix to Chapter 1

A.1 Analytical Results

A.1.1 Euler Equations

Euler equations are derived by following the procedure:

1. Use budget constraint to express consumption choice with state and other choice variables and substitute it for consumption c in the Bellman equation.

2. Rearrange the expected continuation value. $V\left(K^{g(b)}, k^{g(b)}, A'\right)$ is the continuation value if the good (bad) shock is realized and the exogenous shock turns out to be A'. $\left[n \cdot V\left(K^g, k^g, A'\right) + (1-n)\, V\left(K^b, k^b, A'\right)\right]$ is the expected continuation value, given A. Since A' is also stochastic, the expected continuation value, only conditional on A, is

$$E_A\left[n \cdot V\left(K^g, k^g, A'\right) + (1-n)\, V\left(K^b, k^b, A'\right)\right]$$

Therefore, the Bellman equation becomes the following,

$$V\left(K, k, A\right) = \max_{s\geq 0, 1\geq \alpha\geq 0} \left\{ \begin{array}{c} U\left(w + \varphi k - s\right) + \\ \beta \cdot E_A\left[n \cdot V\left(K^g, k^g, A'\right) + (1-n)\, V\left(K^b, k^b, A'\right)\right] \end{array} \right\}$$

3. Assume the interior solution (or $n < 1$) and derive the first-order conditions (from now on, E_A is replaced by E for simplicity)

   the first-order condition with respect to s:

$$U'(c) = \beta \cdot E \left[\begin{array}{c} n \cdot \left(r \cdot \alpha + R \cdot \frac{(1-\alpha)}{n} \right) \cdot V_k (K^g, k^g, A') \\ + (1-n) \cdot r \cdot \alpha \cdot V_k \left(K^b, k^b, A' \right) \end{array} \right]$$

the first-order condition with respect to α:

$$0 = E \left[\begin{array}{c} n \cdot \left(r \cdot s - R \cdot \frac{1}{n} \cdot s \right) \cdot V_k (K^g, k^g, A') \\ + (1-n) \cdot r \cdot s \cdot V_k \left(K^b, k^b, A' \right) \end{array} \right]$$

4. Rearrange the two first-order conditions and the system becomes

$$U'(c) = \beta \cdot R \cdot E \left[V_k \left(K^g, k^g, A' \right) \right]$$

$$U'(c) \left(\frac{1}{r} - \frac{n}{R} \right) = \beta \cdot (1-n) \cdot E \left[V_k \left(K^b, k^b, A' \right) \right]$$

5. Derive the Envelope condition

$$V_k (K, k, A) = U'(c) \cdot \left(\eta \cdot A \cdot K^{\eta-1} + (1-\delta) \cdot \frac{1}{r} \right)$$

6. Use the Envelope condition and update it one period forward. Replace $E \left[V_k (K^g, k^g, A') \right]$ and $E \left[V_k \left(K^b, k^b, A' \right) \right]$ on the right-hand side of the equation system. The equation system becomes two intertemporal equations,

$$U'(c) = \beta \cdot R \cdot E \left[U'(c_g) \cdot \left(\eta \cdot A' \cdot K^{g(\eta-1)} + (1-\delta) \cdot \frac{1}{r} \right) \right]$$

$$U'(c) = \beta \cdot \frac{(1-n)}{\left(\frac{1}{r} - \frac{n}{R} \right)} \cdot E \left[U'(c_b) \cdot \left(\eta \cdot A' \cdot K^{b(\eta-1)} + (1-\delta) \cdot \frac{1}{r} \right) \right]$$

where

$$c_g = c \left(K^g, k^g, A' \right) \text{ and } c_b = c \left(K^b, k^b, A' \right)$$

Note that in the states where $n = 1$, backward inequality holds in the second equation. It corresponds to the case where the financial market is fully diversified.

A.1.2 A Special Case

The analytical solution can be derived from a special case, where $\delta = 1$ and $U(c) = \log(c)$. Using the guess that consumption is a function of aggregate capital and is not dependent on individual capital or, more precisely, consumption is a constant fraction of aggregate output. That is, $c = \tau \cdot A \cdot K^{\eta}$. Moreover, the conjecture of the law of motion of aggregate capital is the same as the individual capital.

1. Replace consumption in the inter-temporal equations with this guess,

$$\frac{1}{c} = \beta \cdot R \cdot E\left[\frac{1}{\tau \cdot A' \cdot K^{g\eta}} \cdot \left(\eta \cdot A' \cdot K^{g(\eta-1)}\right)\right]$$

$$\frac{1}{c} = \beta \cdot \frac{(1-n)}{\left(\frac{1}{r} - \frac{n}{R}\right)} \cdot E\left[\frac{1}{\tau \cdot A' \cdot K^{b\eta}} \cdot \left(\eta \cdot A' \cdot K^{b(\eta-1)}\right)\right]$$

2. Rearrange both equations. Then, the expectation operator drops out,

$$\frac{1}{c} = \beta \cdot R \cdot \frac{\eta}{\tau \cdot K^{g}}$$

$$\frac{1}{c} = \beta \cdot \frac{(1-n)}{\left(\frac{1}{r} - \frac{n}{R}\right)} \cdot \frac{\eta}{\tau \cdot K^{b}}$$

3. Impose the equilibrium condition, $K = k$.
4. Replace k^g and k^b with their law of motion,

$$\frac{1}{c} = \beta \cdot R \cdot \frac{\eta}{\tau \cdot \left(r \cdot \alpha \cdot s + R \cdot \frac{(1-\alpha)}{n} \cdot s\right)}$$

$$\frac{1}{c} = \beta \cdot \frac{(1-n)}{\left(\frac{1}{r} - \frac{n}{R}\right)} \cdot \frac{\eta}{\tau \cdot r \cdot \alpha \cdot s}$$

5. Savings are replaced by $(1-\tau) \cdot A \cdot K^{\eta}$, since savings are also a constant fraction of output,

$$\frac{1-\tau}{\tau} = \beta \cdot R \cdot \frac{\eta}{\tau \cdot \left(r \cdot \alpha + R \cdot \frac{(1-\alpha)}{n}\right)}$$

$$\frac{1-\tau}{\tau} = \beta \cdot \frac{(1-n)}{\left(\frac{1}{r} - \frac{n}{R}\right)} \cdot \frac{\eta}{\tau \cdot r \cdot \alpha}$$

6. Solving for τ and α from this equation system leads to the solution in the text. It is obvious that τ is indeed a constant. The implied investment in each risky security is

$$F(n) = \frac{R - r}{R - r \cdot n} s$$

7. Impose the equilibrium condition, $F^*(n^*) = \frac{D}{1-\gamma}(n^* - \gamma)$, and the equilibrium $n^*(k, A)$ is thus obtained.

A.1.3 The Model with a Deterministic Trend

In this section, a similar model with deterministic trend is provided. One of the critical assumptions is that the minimum size parameter D is growing with the economy at the same growth rate. This assumption captures the idea that the effect of micro-level project indivisibility (or non-convexity of production feasible set) cannot be eliminated by growth alone[1].

I assume that labor productivity grows constantly at the rate of g, that is $\Gamma_t = \Gamma_{t-1}e^g$, (where Γ_t is the labor productivity level in period t). Thus, the production function at period t is

$$Y_t = A_t K_t^{\eta} \cdot (\Gamma_{t-1} L_t)^{(1-\eta)}$$

The system can be normalized by the growth factor Γ_{t-1}. I use the hat version to denote the normalized variables:

$$\widehat{x_t} = \frac{x_t}{\Gamma_{t-1}}$$

I start with the definition of the value function in sequential form,

$$V_t = E_t \sum_{s=t+1}^{\infty} \beta^t u(c_t)$$

1. Rearranging this with hat variables yields

$$V_t = (\Gamma_{t-1})^{(1-\sigma)} \frac{(\widehat{c_t})^{1-\sigma}}{1-\sigma} + \beta \cdot \left(E_t \sum_{s=t+1}^{\infty} \beta^s (\Gamma_{s-1})^{(1-\sigma)} \frac{(\widehat{c_s})^{1-\sigma}}{1-\sigma} \right)$$

2. Divide both sides by $(\Gamma_{t-1})^{(1-\sigma)}$

$$\frac{V_t}{(\Gamma_{t-1})^{(1-\sigma)}} = \frac{(\widehat{c_t})^{1-\sigma}}{1-\sigma} + \frac{1}{(\Gamma_{t-1})^{(1-\sigma)}} \beta \cdot \left(E_t \sum_{s=t+1} \beta^s (\Gamma_{s-1})^{(1-\sigma)} \frac{(\widehat{c_s})^{1-\sigma}}{1-\sigma} \right)$$

[1] Intuitively, one can justify this assumption by observing that the size of the largest project that a developed economy could afford, is much bigger now than its counterpart a hundred years ago.

3. Denote $\widehat{V_t} = \frac{V_t}{(\Gamma_{t-1})^{(1-\sigma)}}$, so that

$$\widehat{V_t} = \frac{(\widehat{c_t})^{1-\sigma}}{1-\sigma} + \frac{1}{(\Gamma_{t-1})^{(1-\sigma)}}\beta \cdot \left(E_t \sum_{s=t+1} \beta^s (\Gamma_{s-1})^{(1-\sigma)} \frac{(\widehat{c_s})^{1-\sigma}}{1-\sigma} \right)$$

4. Use $\widehat{V_{t+1}} = \frac{V_{t+1}}{(\Gamma_t)^{(1-\sigma)}}$, so that

$$\widehat{V_t} = \frac{(\widehat{c_t})^{1-\sigma}}{1-\sigma} + \frac{1}{(\Gamma_{t-1})^{(1-\sigma)}}\beta \cdot \left((\Gamma_t)^{(1-\sigma)} \cdot E_t \widehat{V_{t+1}} \right)$$

5. Rewrite the resource constraint, capital accumulation and portfolio definition with the hat version

$$\widehat{c} + \widehat{s} = w(\widehat{K}, A) + \varphi\left(\widehat{K}, A\right) \cdot \widehat{k}$$

$$\widehat{\phi} = \alpha \cdot \widehat{s} \text{ and } n \cdot \widehat{F} = (1-\alpha) \cdot \widehat{s}$$

$$\widehat{k'} = \begin{cases} r \cdot \alpha \cdot \widehat{s} + (1-\delta)\,\widehat{k} & if\ j > n \text{ with } 1-n \\ r \cdot \alpha \cdot \widehat{s} + R \cdot \frac{(1-\alpha)}{n} \cdot \widehat{s} + (1-\delta)\,\widehat{k} & if\ j \leq n \text{ with } n \end{cases}$$

where $\widehat{\beta} = e^{g(1-\sigma)}\beta$ and it is easily shown that $w(\widehat{K}, A) = w(K, A)$ and $\varphi\left(\widehat{K}, A\right) = \varphi(K, A)$, respectively. The function form of decision rules will be the same as those in the text.

A.2 Algorithm

I take two steps to solve the general equilibrium problem. First, I take an educated guess of the function form of $n(k, A)$ and discretize this function with a two-dimension piecewise linear function, which is increasing in both k and A [2]. The choice of piecewise linear approximation is based on the observation that $n(k, A)$ is increasing and reaches 1, if k and A are sufficiently large. Taking the guess as given, I solve the two functional equations with a root-finding method. I discretize A and k in a two dimensional space with Chebyshev nodes and then interpolate the consumption rule $c(k, A)$ with two dimensional *Chebyshev approximation* with $nk \times nA$ coefficients, where nk and nA are the numbers of collocation points of

[2] I use lower case k in this subsection to represent capital stock. It does not contradict the previous section, since I am solving numerically the case, where the condition $K = k$ is imposed.

capital and productivity level, respectively. Chebyshev approximation is used to take advantage of the high accuracy of the solution. However, if the policy function displays a kink-shape, the scheme may deliver a poor approximation. To avoid this, I must interpolate the portfolio decision $\alpha(k, A)$ with a *shape-preserving piecewise approximation* (with $nk \times nA$ coefficients). The following steps are taken sequentially.

Step 1: Compute the left-hand side value at each collocation point of the $k - A$ mesh;

Step 2: Compute k^b and k^g, according to the law of motion of capital;

Step 3: Compute the right-hand side value at each collocation point of the $k - A$ mesh, which requires computation of the expectation;

Step 4: Compute the expectation using numerical integration with *a modified Tauchen algorithm;*

Step 5: Solve the $2 \times nk \times nA$ nonlinear equation system with Broyden's method and obtain the two policy functions.[3]

Moreover, I set up an outer loop to solve for $n(k, A)$

Step 1: Guess a pseudo function for $n(k, A)$;

Step 2: Take the function as given and solve for $s(k, A)$ and $\alpha(k, A)$ (see above);

Step 3: Compute $F(k, A)$ and $\frac{D}{1-\gamma}(n(k, A) - \gamma)$ at each state;

Step 4: If the difference is below the tolerance level, $n^*(k, A)$ is found;

Step 5: If not, update $n(k, A)$ with *a generalized bisection method;*

Step 6: Return to Step 2 until the equilibrium condition is satisfied in Step 4.

[3]Care has to be taken that strict inequality in the second Euler equation may hold.

Appendix B

Appendix to Chapter 2

B.1 Proofs

In this subsection, we prove the Lemma 1 and Results 1 and 2.

B.1.1 Lemma 1

Proof. Since $c_1 > 0$, it implies,

$$\widetilde{a_1}^{NR} + y_1 - \widetilde{a_2}^{NR} > 0.$$

And $a_1 = 0$ implies $\widetilde{a_1}^{NR} = -h^*$. Moreover, with assumption 1, the following holds,

$$-\widetilde{a_1}^{NR} > y_{2b} + y_1 - y_r.$$

It follows immediately that,

$$\widetilde{a_2}^{NR} < y_r - y_{2b}.$$

□

B.1.2 Result 1

Proof. Given Lemma 1, we now proceed to prove our result that $\widetilde{a_2}^{NR} > \widetilde{a_2}^{R}$.

In the refinancing case, household solves,

$$\max_{c_1}\ u(c_1) + E\left(u\left(c_2^R\right) + u\left(c_3^R\right)\right)$$

such that

$$c_1 = \widetilde{a_1} + y_1 - \widetilde{a_2}^R.$$

And the first order condition reads,

$$\frac{1}{2}\cdot u'\left(\frac{\left(\widetilde{a_2}^R + y_{2g}\right) + y_r}{2}\right) + \frac{1}{2}\cdot u'\left(\frac{\left(\widetilde{a_2}^R + y_{2b}\right) + y_r}{2}\right) - u'\left(a_1 + y_1 - \widetilde{a_2}^R\right) = 0.$$

In the non-refinancing case, household solves,

$$\max_{c_1}\ u(c_1) + E\left(u\left(c_2^{NR}\right) + u\left(c_3^{NR}\right)\right)$$

such that

$$c_1 = \widetilde{a_1} + y_1 - \widetilde{a_2}^{NR}.$$

The first order condition reads,

$$\frac{1}{2}\cdot u'\left(\frac{\left(\widetilde{a_2}^{NR} + y_{2g}\right) + y_r}{2}\right) + \frac{1}{2}\cdot u'\left(\widetilde{a_2}^{NR} + y_{2b}\right) - u'\left(\widetilde{a_1} + y_1 - \widetilde{a_2}^{NR}\right) = 0,$$

where $\widetilde{a_2}$ is an implicit function of $\widetilde{a_1}$, y_{2g}, y_{2b} and y_r, which can be treated as parameters. For convenience, we can define the following two functions:

$$g^{NR}\left(\widetilde{a_2}^{NR}\right) = \frac{1}{2}\cdot u'\left(\frac{\left(\widetilde{a_2}^{NR} + y_{2g}\right) + y_r}{2}\right) + \frac{1}{2}\cdot u'\left(\frac{\left(\widetilde{a_2}^{NR} + y_{2b}\right) + \widetilde{a_2}^{NR} + y_{2b}}{2}\right) - u'\left(\widetilde{a_1} + y_1 - \widetilde{a_2}^{NR}\right)$$

$$g^{R}\left(\widetilde{a_2}^{R}\right) = \frac{1}{2}\cdot u'\left(\frac{\left(\widetilde{a_2}^{R} + y_{2g}\right) + y_r}{2}\right) + \frac{1}{2}\cdot u'\left(\frac{\left(\widetilde{a_2}^{R} + y_{2b}\right) + y_r}{2}\right) - u'\left(\widetilde{a_1} + y_1 - \widetilde{a_2}^{R}\right)$$

$\widetilde{a_2}^{NR}$ and $\widetilde{a_2}^{R}$ are the solutions to $g^{NR}\left(\widetilde{a_2}^{NR}\right) = 0$ and $g^{R}\left(\widetilde{a_2}^{R}\right) = 0$ respectively.

Suppose by contradiction that $\widetilde{a_2}^{R} \geq \widetilde{a_2}^{NR}$. Recall Lemma 1, we know immediately that,

$$\left[u'\left(\frac{\left(\widetilde{a_2}^{R} + y_{2b}\right) + y_r}{2}\right)\right] < \left[u'\left(\frac{\left(\widetilde{a_2}^{NR} + y_{2b}\right) + \widetilde{a_2}^{NR} + y_{2b}}{2}\right)\right],$$

Moreover, it is straightforward that,

$$\begin{aligned}
&\frac{1}{2}\cdot\left[u'\left(\frac{\left(\widetilde{a_2}^{R} + y_{2g}\right) + y_r}{2}\right)\right] - u'\left(a_1 + y_1 - \widetilde{a_2}^{R}\right) \\
&\leq \frac{1}{2}\cdot\left[u'\left(\frac{\left(\widetilde{a_2}^{NR} + y_{2g}\right) + y_r}{2}\right)\right] - u'\left(a_1 + y_1 - \widetilde{a_2}^{NR}\right).
\end{aligned}$$

Combing both of the inequalities, implies that,

$$g^{R}\left(\widetilde{a_2}^{R}\right) < g^{NR}\left(\widetilde{a_2}^{NR}\right) = 0.$$

which is a contradiction to the condition that $g^{R}\left(\widetilde{a_2}^{R}\right) = 0$. Therefore, it has to be true that $\widetilde{a_2}^{R} < \widetilde{a_2}^{NR}$. □

B.1.3 Result 2

Proof. Building on Lemma 1, we can also show that the dispersion of consumption is greater in the traditional mortgage case compared to the refinancing model. First, recall the optimal choices of consumption in the second period for the two different regimes are:

$$c_{2g}^{R} = \frac{\left(\widetilde{a_2}^{R} + y_{2g}\right) + y_r}{2},$$

$$c_{2b}^{R} = \frac{\left(\widetilde{a_2}^{R} + y_{2b}\right) + y_r}{2},$$

$$c_{2g}^{NR} = \frac{\left(\widetilde{a_2}^{NR} + y_{2g}\right) + y_r}{2},$$

and

$$c_{2b}^{NR} = \widetilde{a_2}^{NR} + y_{2b},$$

Prove by contradiction:

$$\frac{\left(\widetilde{a_2}^R + y_{2g}\right) + y_r}{2} - \frac{\left(\widetilde{a_2}^R + y_{2b}\right) + y_R}{2} > \frac{\left(\widetilde{a_2}^{NR} + y_{2g}\right) + y_r}{2} - \widetilde{a_2}^{NR} - y_{2b},$$

which can be reformulated as,

$$\widetilde{a_2}^R + y_{2g} + y_r - \widetilde{a_2}^R - y_{2b} - y_r > \widetilde{a_2}^{NR} + y_{2g} + y_r - 2\widetilde{a_2}^{NR} - 2y_{2b},$$

$$-y_r > -\widetilde{a_2}^{NR} - y_{2b},$$

$$0 > y_r - \widetilde{a_2}^{NR} - y_{2b},$$

which contradicts Lemma 1, which states $y_r - \widetilde{a_2}^{NR} - y_{2b} > 0$ □

B.2 Computational Details

As discussed in the main text, the fact that housing stock is traded with the transaction costs is an important feature of the housing market. Transaction costs induce an "inaction zone"of the household's maximization problem. In other words, there are discontinuities in policy functions and kinks in the value functions and ex ant it is difficult to know where the discontinuities exist. We thus resort to the value function iteration method with discretization of the whole state space, which proves to be very robust. It is well known that the computing time increases exponentially, when discretization of state space increases linearly. We make use of parallel computing techniques to speed up the policy functions computing process.

We follow the algorithm below for solving stationary equilibrium:

1. Take interest rate r as given and compute the implied aggregate capital K and the associated wage rate w.

2. Guess the discount factor β.

3. Solve for the value function in the last period of retired households, then solve value functions recursively backwards for all the other age groups. The associated policy functions are obtained.

4. Compute the stationary distribution of households, given the policy functions from step 3.

5. Given the stationary distribution, compute the aggregate wealth-to-income ratio.

6. If the ratio is consistent with the target, then the open-economy equilibrium is found. If not, go back to step 1 and update β.

Appendix C

Appendix to Chapter 3

C.1 First Order Conditions

The Lagrangian is set up as follows,

$$\begin{aligned} L &= \frac{\left(c^{\gamma}\left(1-l\right)^{1-\gamma}\right)^{1-\sigma}}{1-\sigma} + \beta \cdot e^{g\cdot\gamma\cdot(1-\sigma)} \cdot EV(k', b', z', g') \\ & +\lambda \cdot \left[y + (1-\delta)\,k - \frac{\phi}{2}\left(\frac{k'}{k} \cdot e^{g} - e^{\mu_g}\right)^2 k - b + qb' \cdot e^{g} - c - k' \cdot e^{g} \right] \end{aligned}$$

The household chooses consumption optimally and it gives Equation (3.6) in the text,

$$\left[c^{\gamma}\left(1-l\right)^{(1-\gamma)}\right]^{-\sigma} \gamma c^{\gamma-1}\left(1-l\right)^{(1-\gamma)} = \lambda.$$

Similarly, he chooses labor supply optimally, which results in the following condition,

$$\left[c^{\gamma}\left(1-l\right)^{(1-\gamma)}\right]^{-\sigma} \left(1-\gamma\right) c^{\gamma}\left(1-l\right)^{-\gamma} = \lambda \alpha e^{z} k^{1-\alpha} e^{g}\left(e^{g} l\right)^{\alpha-1}. \tag{C.1}$$

Further, Equation (3.6) and (C.1) imply Equation (3.7) in the text,

$$\frac{c}{(1-l)} = \frac{\gamma}{(1-\gamma)} y_L.$$

He chooses capital holding, given that he takes into account the adjustment cost,

$$\beta \cdot e^{g\cdot\gamma\cdot(1-\sigma)} \cdot EV_{k'}(k', b', z', g') + \lambda \cdot \left[-\phi \cdot e^{g} \cdot \left(\frac{k'}{k} \cdot e^{g} - e^{\mu_g} \right)^2 - e^{g} \right] = 0. \quad \text{(C.2)}$$

Finally, he also chooses the debt level, so that

$$\beta \cdot e^{g\cdot\gamma\cdot(1-\sigma)} \cdot EV_{b'}(k', b', z', g') + \lambda \cdot q \cdot e^{g} = 0. \quad \text{(C.3)}$$

In order to solve for both $V_{k'}(\cdot)$ and $V_{b'}(\cdot)$, the Envelop theorem is applied, which results in Equation (C.4) and (C.5), respectively.

$$V_k = \lambda \cdot \left[y_k + (1-\delta) - \frac{\phi}{2} \left[\left(\frac{k'}{k} \cdot e^{g} - e^{\mu_g} \right)^2 - 2 \left(\frac{k'}{k} \cdot e^{g} - e^{\mu_g} \right) \frac{k'}{k} \cdot e^{g} \right] \right]. \quad \text{(C.4)}$$

$$V_b = -\lambda. \quad \text{(C.5)}$$

Update Equation (C.4) and combine it with Equation (C.2), the Equation (3.9) is obtained. Similarly, Equation (3.8) can be found with Equation (C.5) and (C.3).

C.2 Steady State Information

In this subsection, all the variables in steady state are denoted with *. Based on Equation (3.1), the interest rate equals the international interest rate, $r = r^*$, in the steady state. And therefore, the price of bond at the steady state, q^*, is,

$$q^* = \frac{1}{1+r^*}.$$

Moreover, since the marginal productivity of capital should equate the international interest rate, it is straightforward to solve for the income-capital ratio in steady state from production function, Equation (3.2),

$$\frac{y^*}{k^*} = \frac{r^*}{1-\alpha}.$$

With the budget constraint, Equation (3.5), the consumption-income ratio can be derived,

$$\frac{c^*}{y^*} = 1 + (1 - \delta - e^{\mu_g}) \frac{k^*}{y^*} + (q^* \cdot e^{\mu_g} - 1) \frac{b^*}{y^*}.$$

Labor supply in the steady state is solved for with the intra-temporal relation between labor and consumption choice, Equation (3.7),

$$l^* = \left(1 + \frac{c^*}{y^*} \frac{(1-\gamma)}{\gamma\alpha}\right)^{-1}.$$

Using production function again, the steady state information on capital is implied,

$$k^* = \left(\frac{k^*}{y^*}\right)^{\frac{1}{\alpha}} \cdot e^{u_g} \cdot l^*$$

It is straightforward to compute steady state information on output, y^*, consumption, c^*, leisure, n^*, net export, nx^*, and savings, s^*.

C.3 Log-Linearization

I log-linearize the whole equation system Υ around the non-stochastic steady state (found in Appendix C.2) to obtain a linear difference equation system in the logs of endogenous state variables, k and b, choice variables, y, c, n, l, nx and s, as well as the exogenous productivity shocks, z and g. Formally, let x^* be the steady state value of variable x and $\widehat{x}$ the log-deviation of variable x from steady state x^*,

$$\widehat{x} = \frac{(x - x^*)}{x^*} \approx log\,(x) - log\,(x^*)\,.$$

The Cobb-Douglas production function (Equation (3.2)) is linear in logs and therefore it requires no approximation:

$$\widehat{y} = z + (1-\alpha) \cdot \widehat{k} + \alpha \cdot \left(\widehat{e^g} + \widehat{n}\right).$$

The interest rate function (Equation (3.1)) is approximated by,

$$\widehat{q} = -\psi \cdot q^* \cdot b^* \cdot \widehat{b'}.$$

The budget constraint (Equation (3.5)) is replaced by the following log-linear version,

$$0 = y^*\widehat{y} - \widehat{b}b^* + q^*b^*e^{\mu_g}\left(\widehat{b'} + \widehat{q} + \widehat{e^g}\right) - c^*\widehat{c} - \widehat{x}X^*,$$

where recall the definition of gross investment,

$$\widehat{x}X^* = k^* \left(e^{\mu_g}\widehat{k'} - (1-\delta)\,\widehat{k} + \widehat{e^g}\right).$$

The next step is to log-linearize the first order conditions which are necessary to characterize the solution of the system Υ. The intra-temporal choice between labor and consumption (Equation (3.7)) is linearized by

$$0 = \widehat{y} - \widehat{n} - \widehat{c} + \widehat{l}.$$

And moreover, the pricing formula for bond price (Equation (3.8)) and the inter-temporal condition (Equation (3.9)) are approximated by,

$$\begin{aligned} 0 \;=\;& [\gamma(1-\sigma)-1]\,E\widehat{c'} + [(1-\sigma)(1-\gamma)]\,E\widehat{l'} \\ & + [\gamma(1-\sigma)-1]\,\widehat{e^g} - [\gamma(1-\sigma)-1]\,\widehat{c} \\ & - [(1-\sigma)(1-\gamma)]\,\widehat{l} - \widehat{q}, \end{aligned}$$

and

$$\begin{aligned} 0 \;=\;& [\gamma(1-\sigma)-1]\,E\widehat{c'} + [(1-\sigma)(1-\gamma)]\,E\widehat{l'} + [\gamma(1-\sigma)]\,E\widehat{e^{g'}} \\ & + \left[(1-\alpha)\frac{y^*}{k^*}\right]\left[\beta e^{\mu_g\cdot(\gamma(1-\sigma)-1)}\right]E\widehat{y'} + \left[\beta e^{\mu_g\cdot(\gamma(1-\sigma)-1)}\right]\phi E\widehat{k''} \\ & - \left[\beta e^{\mu_g\cdot(\gamma(1-\sigma)-1)}\left((1-\alpha)\frac{y^*}{k^*} + \phi e^{2\cdot\mu_g}\right) + \phi e^{\mu_g}\right]\widehat{k'} - [\gamma(1-\sigma)-1]\,\widehat{c} \\ & - [(1-\sigma)(1-\gamma)]\,\widehat{l} + [\phi e^{\mu_g} + 1 - \gamma(1-\sigma)]\,\widehat{e^g} + \phi e^{\mu_g}\cdot\widehat{k} \end{aligned}$$

It is straightforward to log-linearize the rest of the equation system, the three identities which characterize labor-leisure, net export and savings.

$$\widehat{y} = \widehat{Z} + (1-\alpha)\,\widehat{k} + \alpha\left(\widehat{G} + \widehat{n}\right)$$

$$\widehat{nx}\,(nx)^* \;=\; (1-(nx)^*)\,\widehat{y} - \frac{X^*}{y^*}\widehat{X} - \frac{c^*}{y^*}\widehat{c}$$

$$s^*\widehat{s} = \widehat{y}y^* - \widehat{c}c^*$$

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